Additions to Second Edition Errata and Comments

May 5, 2002

We thank Mohamed Elhamdadi, Chris Hruska, and Dick Palas for pointing out errors.

Page xv 16th line from bottom: "some of the material," not "some of material."

Page 4 Margin note about Bowditch: "American Bowditch," not "America Bowditch."

Page 5 There are some inconsistencies of notation. In future editions we will write Equation 0.2.2 as

$$\text{The opposite of} \quad (\forall\, x)P(x) \quad \text{is} \quad (\exists x) \text{ not } P(x).$$

(But if we had something more complicated than $P(x)$ we would put it in parentheses.)

Most mathematicians avoid the symbolic notation, instead writing out quantifiers in full. But when there is a complicated string of quantifiers, they often use the symbolic notation to avoid ambiguity.

Page 7 Note that it follows from the definition of $\subset$ that the empty set ϕ is a subset of every set.

Page 18 Definition 0.5.1: The least upper bound is also known as the supremum. Definition 0.5.2: The great lower bound is also known as the infimum.

Page 21 In the first line of the proof of Theorem 0.5.8, the summmand should be in parentheses: $\sum_{n=1}^{\infty}(a_n + |a_n|)$

Exercise 0.5.1: Exercise 1.6.11 repeats this exercise, with hints.

Page 24 Second full paragraph: $\mathcal{P}(E)$ is called the *power set* of E.

Page 40 In the margin note beginning "The trivial subspace," the **0** should be $\vec{\mathbf{0}}$.

Page 45 Sentence immediately before Definition 1.2.4: "I If" should be "If."

Page 61 In the first paragraph of the remark, the mention of "feedback" is incorrect. Feedback is compatible with linearity. The end of the paragraph, beginning with "Nor do linear transformations allow for feedback," has been rewritten as follows:

Nor does a linear model of the "price transformation" allow for the possibility that if you buy more you will get a discount for quantity, or that if you buy even more you might create scarcity and drive prices up. The failure to take such effects into consideration is a fundamental weakness of all models that linearize mappings and interactions.

Page 79 Figure 1.4.9: $\mathbf{h}$ should be h, the height of the parallelogram.

Page 94 Third line: "an incontestably correct definition," not "a incontestably correct definition."

Page 100 Theorem 1.5.23: We should have specified that U is a subset of $\mathbb{R}^n$. In Equation 1.5.35, we should have written $(h\mathbf{f})(\mathbf{x})$, not $h\mathbf{f}(\mathbf{x})$.

Page 105 Last margin note: "Newton's method," not "Newton's method's."

Page 126 It is incorrect to ascribe the motions of a pendulum to feedback.

Page 147 Last line: following the equation, we should perhaps add "i.e., for every $\vec{\mathbf{h}} \in \mathbb{R}^n$ we have $[\mathbf{Df}(\mathbf{a})]\vec{\mathbf{h}} = \mathbf{f}(\vec{\mathbf{h}})$."

Page 157 Parentheses should be added to Equation 1.9.15:

$$\lim_{x\to 0}\left(\frac{1}{2} + 2x\sin\frac{1}{x}\right) = \frac{1}{2}.$$

Page 161 Exercise 1.9.1 is identical to Example 1.9.4.

Page 165 Exercise 1.22 is identical to Exercise 1.5.19, which in any case should be in Section 1.6.

Page 178 Margin note: The vector $\vec{\mathbf{b}}$ does not contain the solutions.

Page 179 In the remark, we mention linear independence prematurely; it is not discussed until Section 2.4.

Page 205 Exercise 2.4.11 should be with the exercises for Section 2.5.

Page 206 Part (a) of Exercise 2.4.13 was poorly stated. It should be:

(a) For $n = 1, n = 2, n = 3$, write the system of linear equations which the $a_{0,n}, \dots, a_{n,n}$ must satisfy so that the integral of 1 is exact, the integral of x is exact, and so on, until you get to x^n.

Page 212 Corollary 2.5.11: Rather than "i.e., if the kernel is zero" it would be better to say, "i.e., if the kernel has dimension 0."

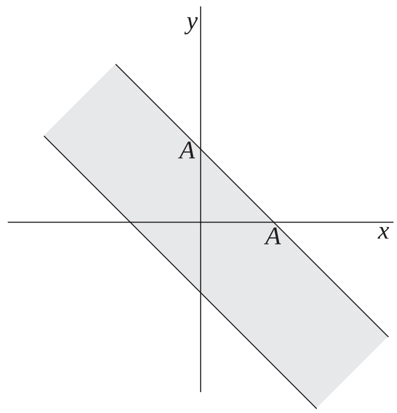
Page 226 Fourth line of Example 2.6.3: a space is needed between "Example 2.6.2" and "and."

Page 236 After the displayed equation in Exercise 2.6.5, $\Phi^{-1}_{\{\underline{\mathbf{v}}\}}$ should be $\Phi_{\{\underline{\mathbf{v}}\}}$: "so that

$$\Phi_{\{\underline{\mathbf{v}}\}}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}."$$

Page 237 Exercise 2.6.11: A_a should be A.

Page 244 The way Figure 2.7.5 is drawn, the shaded strip seems to end in quadrants 2 and 4; actually, it is infinite. The following may suggest reality better:



Page 249 In the bottom margin note, we discuss the importance of making sure both sides of an equation have the same units. In chemical engineering, fluid mechanics, etc., this is called "dimensional analysis."

Page 256 The margin note about Exercise 2.23 belongs on page 288.

Exercise 2.7.11 is missing part (b):

(b) Prove that this Newton's method converges.

Page 261 On the second line of Example 2.8.9 we say that the norm is $\frac{1+\sqrt{5}}{2}$; in Equation 2.8.8 we compute the norm as $\sqrt{\frac{3+\sqrt{5}}{2}}$. Both, of course, are correct, since

$$\sqrt{\frac{3+\sqrt{5}}{2}} = \sqrt{\frac{6+2\sqrt{5}}{4}} = \frac{1+\sqrt{5}}{2}.$$

Page 265 First sentence after Theorem 2.9.2: Exercise A.7.1, not 7.1.

Page 271 At the end of the first paragraph after Figure 2.9.6: "look at condition (3a) of the theorem" should be "look at condition (1) of the theorem." In the next paragraph, "condition (3b) is more delicate" should be "condition (2) is more delicate."

Page 273 In the first line after Equation 2.9.18, the reference is to the wrong equation. "Next we need to compute the Lipschitz ratio M (Equation 2.9.24)" should be "Next we need ... (Equation 2.9.11)."

Pages 277, 278 The margin note about Equation 2.9.30 (page 277) should be on page 278.

Page 280 The second margin note is completely false; we have no idea what we were thinking of. Using the second partial derivative method in Example 2.9.15 is perfectly possible and gives a Lipschitz ratio of $2\sqrt{3}$.

Page 288 The margin note on page 256 about Exercise 2.23 belongs on this page.

Page 351 Exercise 3.5.1: "and finally the terms in x," not "and finally the terms in y."

Page 468 At present, Theorem 4.7.5 requires f to be integrable. In a future edition, we will change Theorem 4.7.5 to something like

Theorem 4.7.5 (Integrals using arbitrary pavings) . *Let $X \subset \mathbb{R}^n$ be a bounded subset, and $\mathcal{P}_N$ be a nested partition of X. Suppose the boundary ∂X satisfies $\text{vol}_n(\partial X) = 0$. Then $f : \mathbb{R}^n \to \mathbb{R}$ is integrable if and only if the upper and lower limits using the nested partition are equal:*

$$\lim_{N\to\infty} U_{\mathcal{P}_N}(f) = \lim_{N\to\infty} L_{\mathcal{P}_N}(f). \tag{4.7.4}$$

In that case, they are both equal to

$$\int_X f(\mathbf{x})\,|d^n\mathbf{x}|. \tag{4.7.5}$$

This will solve some problems with the current proof of Theorem 4.9.1.

Page 488 In the equation following Equation 4.9.12, the left-hand side should be

$$U_{T(\mathcal{D}_N)}(\chi_{T(A)}) - L_{T(\mathcal{D}_N)}(\chi_{T(A)});$$

the upper and lower sums are with respect to the nested partition $T(\mathcal{D}_N)$.

Page 602 The solution to Exercise 6.4.6 uses the formula

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Justifying this formula uses three statements taught in one-variable calculus and the fact (Proposition 1.5.34) that absolute convergence implies convergence. The three statements are the expression of $\sin t$, $\cos t$, and e^t, for t real, in terms of power series:

$$\begin{aligned}\sin t &= t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots\\ \cos t &= 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots\\ e^t &= 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots .\end{aligned} \tag{1}$$

First, let us show that for a complex number z, we can define e^z by the power series

$$e^z = 1 + z + \frac{z^2}{2!} + \cdots .$$

We know it is true in the special case where z is real. We need to check that the series converges. The series $1 + |z| + \left|\frac{z^2}{2!}\right| + \cdots$ converges, since (by Equation (1): $|z|$ is a real number)

$$\sum_{k=0}^{\infty} \left|\frac{z^k}{k!}\right| = \sum_{k=0}^{\infty} \frac{|z|^k}{k!} = e^{|z|}$$

converges. So Proposition 1.5.34 says that $\sum_{k=0}^{\infty} \frac{z^k}{k!}$ converges.

Now write

$$\begin{aligned}
\cos t + i \sin t &= \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots\right) + i\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \right) \\
&= \left(1 + \frac{(it)^2}{2!} + \frac{(it)^4}{4!} + \frac{(it)^6}{6!} + \cdots\right) + \left(it + \frac{(it)^3}{3!} + \frac{(it)^5}{5!} + \right) \\
&= 1 + it + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \frac{(it)^4}{4!} + \cdots = e^{it}.
\end{aligned}$$