

Additions to Second Edition Errata and Comments

December 5, 2002

Once again, many thanks to Phillip Burns and Harry Hirsch for pointing out the errors below.

Page 93 In Equation 1.5.13, U should be \bar{U} .

Page 137 Equation 1.7.37 should have some parentheses:

$$\lim_{h \rightarrow 0} \frac{1}{h} \left(\overbrace{(1+h)(1+2h) \sin\left(\frac{\pi}{2} + h\right)}^{\mathbf{f}(\mathbf{a}+h\bar{\mathbf{v}})} - \overbrace{(1 \cdot 1 \cdot \sin\frac{\pi}{2})}^{\mathbf{f}(\mathbf{a})} \right).$$

Page 246 In Example 2.7.11, we use a different order for the subscripts of c than that given in Proposition 2.7.10. To make the text consistent, $c_{2,2,1}$ should be $c_{1,2,2}$ and $c_{1,1,2}$ should be $c_{2,1,1}$:

$$|D_2 D_2 \mathbf{f}_1| \leq 3A = \underbrace{c_{1,2,2}}_{\text{bound for } |D_2 D_2 \mathbf{f}_1|} \quad \text{and} \quad |D_1 D_1 \mathbf{f}_2| \leq 3A = \underbrace{c_{2,1,1}}_{\text{bound for } |D_1 D_1 \mathbf{f}_2|}$$

with all others 0, so

$$\sqrt{c_{1,2,2}^2 + c_{2,1,1}^2} = 3A\sqrt{2}. \tag{2.7.40}$$

Page 246 Four lines from the bottom— one reader wondered whether “blunderbuss” was “a new word from generation X”. Our dictionary defines a blunderbuss as an “old-fashioned, short gun with large bore and flaring mouth, used for scattering shot at close range”. It will hit a big target, but is not precise.

Page 253 As on page 246, the order of subscripts for c is wrong in three places at the bottom of the page. Below, the starred entries have been corrected:

$$\begin{aligned} \sup |D_1 D_1 f_1| &\leq 3 = c_{1,1,1} & * \sup |D_1 D_1 f_2| &= 0 = c_{2,1,1} \\ \sup |D_1 D_2 f_1| &\leq 1 = c_{1,2,1} & * \sup |D_1 D_2 f_2| &= 0 = c_{2,2,1} \\ * \sup |D_2 D_2 f_1| &\leq 1 = c_{1,2,2} & \sup |D_2 D_2 f_2| &= 2 = c_{2,2,2}. \end{aligned}$$

Page 270 We never proved Equation 2.9.13! Moreover, it is wrong, which shows how dangerous it is to omit proofs. The correct equation is

$$R_1 = R|L^{-1}|^2 \left(\sqrt{|L|^2 + \frac{2}{|L^{-1}|^2}} - |L| \right).$$

Proof. Suppose $|\mathbf{x} - \mathbf{x}_0| < R_1$. Then

$$|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{x}_0)| \leq |\mathbf{x} - \mathbf{x}_0| \sup \|[\mathbf{Df}(\mathbf{x})]\| \leq R_1 \sup \|[\mathbf{Df}(\mathbf{x})]\|.$$

We find a bound for $\|[\mathbf{Df}(\mathbf{x})]\|$:

$$\|[\mathbf{Df}(\mathbf{x})] - [\mathbf{Df}(\mathbf{x}_0)]\| = \|[\mathbf{Df}(\mathbf{x})] - L\| \stackrel{\text{Eq. 2.9.11}}{\leq} \frac{1}{2R|L^{-1}|^2} |\mathbf{x} - \mathbf{x}_0| \leq \frac{R_1}{2R|L^{-1}|^2}$$

so

$$\|[\mathbf{Df}(\mathbf{x})]\| \leq |L| + \frac{R_1}{2R|L^{-1}|^2}, \quad \text{i.e.,} \quad \sup \|[\mathbf{Df}(\mathbf{x})]\| = |L| + \frac{R_1}{2R|L^{-1}|^2}.$$

Therefore (remember that R is the radius of V , the domain of \mathbf{g}) we want to find the largest R_1 satisfying

$$R \geq \left(|L| + \frac{R_1}{2R|L^{-1}|^2} \right) R_1.$$

The right-hand side is 0 when $R_1 = 0$ and then increases as R_1 increases, so we want the largest value of R_1 for which the inequality is an equality. Thus we want to solve the quadratic equation

$$R_1^2 + 2R|L^{-1}|^2 |L|R_1 - 2R^2|L^{-1}|^2 = 0,$$

which gives

$$R_1 = R|L^{-1}|^2 \left(-|L| + \sqrt{|L|^2 + \frac{2}{|L^{-1}|^2}} \right).$$

Page 308 Second line of the remark, “it may look as though”, not “it may looks as though”.

Page 392 The fourth line of Equation 3.8.68 should be

$$= \left(-(\kappa(s(t)))^2 (s'(t))^3 + s'''(t) \right) \vec{\mathbf{t}}(s(t))$$

Page 437 In the last line of the proof of Theorem 4.4.3, the second equation should be

$$\sum_{i,j} \text{vol } B_{i,j} \leq \epsilon$$

(not $\text{vol } X_1 \cup X_2 \cup \dots \leq \epsilon$).

Page 438 p.438 Line 10: Example 4.3.3, not 4.4.2:

... unlike the function of Example 4.3.3, which, as far as we know, is only a pathological example, devised to test the limits of mathematical statements.

Page 534 In Equation 5.2.5, the sum should start at $n = 1$ not $= 1$. On the righthand sides of Equations 5.2.5 and 5.2.6, the denominator should be 2^{N-1} , not 2^{N-2} .

Page 549 In the second line of Equation 5.3.45, three closing parentheses aren't opened. The line should be

$$= \det \begin{bmatrix} 1 + (D_1f)^2 & (D_1f)(D_2f) & (D_1f)(D_3f) \\ (D_1f)(D_2f) & 1 + (D_2f)^2 & (D_2f)(D_3f) \\ (D_1f)(D_3f) & (D_2f)(D_3f) & 1 + (D_3f)^2 \end{bmatrix}$$

Appendix A.8 The proof is not as clear as it should be as to why the root found by Newton's method is unique in all of W_0 and not just in U_0 . This question is addressed by part (3) of the proof of the *inverse* function theorem, which refers to Remark A5.5 on page 688. Since we treat the implicit function theorem as a special case of the inverse function theorem, this is relevant. In any future editions we plan to put the content of Remark A5.5 in Section 2.7, perhaps immediately after the statement of the Kantorovich theorem.

Inside back cover The "useful formulas: trigonometry" would be more useful if they were all correct! Sorry! The fourth and fifth formulas should be

$$\cos \alpha = \sin(\pi/2 - \alpha) \quad \text{and} \quad \sin \alpha = \cos(\pi/2 - \alpha).$$

(For the formula for sine, it doesn't make a difference, but for cosine, it does.)