

Corrections to the book *Algebraic Topology* by Allen Hatcher

Some of these are more in the nature of clarifications than corrections. Many of the corrections have already been incorporated into later printings of the book.

- Chapter 0, page 9. In the next-to-last paragraph delete the sentence “This viewpoint makes it easy to see that the join operation is associative.” Also, in the sentence preceding this one, change the word “regarded” to “constructed”. Set-theoretically it is true that join is associative, but there are examples where the topologies on $(X * Y) * Z$ and $X * (Y * Z)$ can be different. This is another instance of how mixing product and quotient constructions can lead to bad point-set topological behavior. For CW complexes the issue can be avoided by using CW topologies, as in the first paragraph at the top of the next page.

- Chapter 0, page 9, line -11. Replace $0t_i$ by $0x_i$.

- Chapter 0, page 15, Example 0.15. If you have an early version of this chapter with no figure for this example, then in the next-to-last line of this paragraph change “the closure of $X - N$ ” to “ $X - h(M_f - Z)$ ”. [This paragraph was rewritten for later versions, making this correction irrelevant.]

- Chapter 0, page 17. The fourth line should say that (Y, A) has the homotopy extension property, rather than (X, A) . Also, in the next paragraph there are two places where $k_{tu} : A \rightarrow A$ should be changed to $k_{tu} : A \rightarrow X$, in the fourth and twelfth lines following the displayed formula for k_t .

- Chapter 0, page 17. In the third-to-last line, $f_1 : X \rightarrow X$ should be $f_1 : X \rightarrow Y$. Also, on the seventh-to-last line it might be clearer to say “Viewing k_{tu} as a homotopy of $k_t|_A$ ”

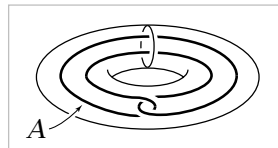
- Chapter 0, page 20, Exercise 26, third line: Change (X, A) to (X_1, A) .

- Chapter 0, page 20, Exercise 27. To avoid point-set topology difficulties, assume that f is not just surjective but a quotient map. Here is a more general version of this exercise: Given a pair (X, A) and a homotopy equivalence $f : A \rightarrow B$, show that the natural map $X \rightarrow B \sqcup_f X$ is a homotopy equivalence if (X, A) satisfies the homotopy extension property.

- §1.1, page 30, line 14. Change “paths lifting the constant path at x_0 ” to “paths lifting constant paths”

- §1.1, page 32, third paragraph. The reference should be to Corollary 2.15, not 2.11.

- §1.1, page 39, Exercise 16(c). In case it’s not clear, the circle A is supposed to be the dark one in the figure, in the interior of the solid torus.



- §1.2, page 46, sixth line from bottom. Repeated “the” — delete one.

- §1.2, page 53, Exercise 5. Part (b) is simply wrong, and should be deleted.
- §1.2, page 54, Exercise 15. It should be specified that if the triangle T has vertices P, Q, R , then the three edges are oriented as PQ, PR, QR .
- §1.2, page 55, line 1. A comment: the reduced suspension depends on the choice of basepoint, so the statement is that C is the reduced suspension of CX with respect to a suitable choice of basepoint.
- §1.3, page 56, second paragraph. A comment about the definition of a covering space: The way that $p^{-1}(U_\alpha)$ could be empty is that it could be the union of an empty collection of open sets homeomorphic to U_α .
- §1.3, page 57, third-to-last line. Change Koenig to König, to agree with the spelling in the Bibliography and in the original source itself.
- §1.3, page 65, line 12. Change “cover space” to “covering space”
- §1.3, page 69, second and third lines of last paragraph. It should say “assuming that X is path-connected, locally path-connected, and semilocally simply-connected”.
- §1.3, page 79, Exercise 3. Add the hypothesis that the covering space map $p: \tilde{X} \rightarrow X$ is surjective.
- §1.3, page 79, Exercise 8. The reference should be to Exercise 11 in Chapter 0, not Exercise 10.
- §1.3, page 82, Exercise 33. Change the ℓ in the fourth line to d .
- §2.1, page 121, line -9. The equations should read $S([w_0]) = w_0(S\partial[w_0]) = w_0(S([\emptyset])) = w_0([\emptyset]) = [w_0]$. [This is corrected in later printings.]
- §2.1, page 125, Example 2.23. Each occurrence of $H_n(S^n)$ in this example should have a tilde over the H .
- §2.1, page 127. In the discussion of naturality two-thirds of the way down the page the maps α, β , and γ should be assumed to be chain maps.
- §2.1, page 129, next-to-last paragraph. In each of the first two lines of this paragraph there is a c that should be c' .
- §2.2, page 134. The notion of degree for maps $S^n \rightarrow S^n$ is not very interesting when $n = 0$, so it may be best to exclude this case from the definition to avoid having to think about trivialities and whether H_n should be \tilde{H}_n or not.
- §2.2, page 135, last line. Add the nontriviality condition $n > 0$, to guarantee that the groups $H_n(S^n)$ in the diagram on the next page are \mathbb{Z} .
- §2.2, page 137, line 6. Change the word “stretching” to “shrinking.”
- §2.2, page 152. In the exact sequence at the top of the page delete the final 0 and the arrow leading to it.

■ §2.2, page 156, Exercise 13. The second half of part (b) should say that the only subcomplex $A \subset X$ for which the quotient map $X \rightarrow X/A$ is a homotopy equivalence is the trivial subcomplex consisting of the 0-cell alone.

■ §2.3, page 164. There is something wrong with the syntax of the long sentence beginning on line 8 of this page, the second example of a functor. The simplest correction would be to change the word “assigns” to “assigning” in line 8. Perhaps a better fix would be to break this long sentence into two sentences by putting a period at the end of line 9 and then starting a new sentence on line 10 with “This is a functor from the category ...”.

■ §2.B, page 173. In the second paragraph after Theorem 2B.5 the historical comments are in need of corrections. Frobenius’ theorem needs the hypothesis that the division algebra has an identity element, and Hurwitz only proved that the condition $|ab| = |a||b|$ implies the dimension must be 1, 2, 4, or 8. Here is a revised version of this paragraph:

The four classical examples are \mathbb{R} , \mathbb{C} , the quaternion algebra \mathbb{H} , and the octonion algebra \mathbb{O} . Frobenius proved in 1877 that \mathbb{R} , \mathbb{C} , and \mathbb{H} are the only finite-dimensional associative division algebras over \mathbb{R} with an identity element. If the product satisfies $|ab| = |a||b|$ as in the classical examples, then Hurwitz showed in 1898 that the dimension of the algebra must be 1, 2, 4, or 8, and others subsequently showed that the only examples with an identity element are the classical ones. A full discussion of all this, including some examples showing the necessity of the hypothesis of an identity element, can be found in [Ebbinghaus 1991]. As one would expect, the proofs of these results are algebraic, but if one drops the condition that $|ab| = |a||b|$ it seems that more topological proofs are required. We will show in Theorem 3.20 that a finite-dimensional division algebra over \mathbb{R} must have dimension a power of 2. The fact that the dimension can be at most 8 is a famous theorem of [Bott & Milnor 1958] and [Kervaire 1958]. See §4.B for a few more comments on this.

■ §2.B, page 176, Exercise 3. A better hint would be to glue two copies of (D^n, D) to the two ends of $(S^{n-1} \times I, S \times I)$ to produce a k -sphere in S^n and look at a Mayer-Vietoris sequence for the complement of this k -sphere. (The hint originally given leads to problems with the point-set topology hypotheses of the Mayer-Vietoris sequence.)

■ §2.C, page 180. In the line preceding the proof of 2C.3 the S^3 should be S^4 . Also, in the line above this the reference should be to Example 4L.4 rather than to an exercise in section 4K.

■ §2.C, page 180. The last sentence on this page continuing onto the next page is somewhat unnecessary since the fact that K is a subdivision of L implies that its simplices have diameter less than $\varepsilon/2$.

■ Introduction to Chapter 3, page 187. In the fourth-to-last line change “homology group” to “cohomology group”.

- §3.1, page 198, line 20. There are two missing φ 's. It should read $\varphi(\partial\sigma) = \varphi(\sigma(v_1)) - \varphi(\sigma(v_0)) = 0$.
- §3.1, page 202 line 5. Change $H^n(X, A)$ to $H^n(X, A; G)$.
- §3.2, page 213, third paragraph, third line. Change $P^n - \{0\}$ to $P^n - \{p\}$.
- §3.2, page 215. In the statement of Theorem 3.14 change “with” to “when”.
- §3.2, page 217, sixth to last line. Change “a special case of the former if $2 \neq 0$ in R ” to “a consequence of the former if R has no elements of order 2”.
- §3.2, page 218, last line of second paragraph: Change the first Y to X , so that the tensor product becomes $H^*(X; R) \otimes_R H^*(Y; R)$.
- §3.2, page 221, line 9. The strict inequality $n > i$ could be changed to $n \geq i$, although this is not important for the argument being made.
- §3.2, page 227, first sentence. The reference to the 1980 paper of Adams and Wilkerson is incorrect. In fact the proof of this fundamental result has only been completed recently in work of K. Andersen and J. Grodal that has yet to be published.
- §3.2, page 228. The algebraic problem referred to at the end of the first paragraph on this page has been solved. The answer is what one would hope: The simplicial complex C_X is uniquely determined by the cohomology ring $H^*(X; \mathbb{Z})$. In fact this is true with \mathbb{Z}_2 coefficients. A similar result holds also in the situation mentioned in the following paragraph, so a subcomplex of a product of n copies of $\mathbb{C}P^\infty$ is uniquely determined by its cohomology ring, up to permutation of the factors (and deletion of a $\mathbb{C}P^\infty$ factor if none of its positive-dimensional cells are used). The reference is Theorem 3.1 in J. Gubeladze, The isomorphism problem for commutative monoid rings, J. Pure Appl. Alg. 129 (1998), 35-65.
- §3.2, page 228, Example 3.24. Change Macauley to Macaulay (3 times). Also in the Index, page 540, under Cohen-Macaulay.
- §3.2, page 229, Exercise 4. The reference should be to Exercise 3 in §2.C.
- §3.2, page 229, Exercise 5. Change this to: Show the ring $H^*(\mathbb{R}P^\infty; \mathbb{Z}_{2k})$ is isomorphic to $\mathbb{Z}_{2k}[\alpha, \beta]/(2\alpha, 2\beta, \alpha^2 - k\beta)$ where $|\alpha| = 1$ and $|\beta| = 2$. [Use the coefficient map $\mathbb{Z}_{2k} \rightarrow \mathbb{Z}_2$ and the proof of Theorem 3.12.]
- §3.2, page 230. In the next to last line of Exercise 14 the exponent on α should be $2n + 1$ instead of $n + 1$.
- §3.2, page 230, Exercise 17. This can in fact be done by the same method as in Proposition 3.22, although the details are slightly more complicated. For a write-up of this, see the webpage for the book under the heading of Revisions.
- §3.3, page 234, line 7. Change “neighborhood of A ” to “neighborhood of the closure of A ”.

- §3.3, page 236. In the sixth line of the longish paragraph between Theorem 3.26 and Lemma 3.27, change the phrase “for B any open ball in M ” to “for B any open ball in M containing x .”
- §3.3, page 239, next-to-last line: Change “ $(k - \ell)$ -simplex” to “ $(k - \ell)$ -chain”
- §3.3, page 241. In the ninth-to-last line change “cycle” to “cocycle”.
- §3.3, page 242. In line 5 of the subsection *Cohomology with Compact Supports* change “chain group” to “cochain group.”
- §3.3, page 245. At the end of the first paragraph on this page it is stated that inclusion maps of open sets are proper maps, but this is not generally true. A proper map $f : X \rightarrow Y$ does induce maps $f^* : H_c^i(Y; G) \rightarrow H_c^i(X; G)$, but the proof of Poincaré duality uses induced maps of a different sort going in the opposite direction from what is usual for cohomology, maps $H_c^i(U; G) \rightarrow H_c^i(V; G)$ associated to inclusions $U \hookrightarrow V$ of open sets in the fixed manifold M .
- §3.3, page 245. In the diagram in the middle of the page the two vertical arrows are pointing in the wrong direction in the first printing of the book. This was corrected in the second printing.
- §3.3, page 248. In the next-to-last line of item (1) in the proof of Poincaré Duality, change “the cocycle taking” to “a cocycle φ taking”
- §3.3, page 249, line 12. Change H^{i-1} to H^{i+1} .
- §3.3, page 249. In the line above the commutative diagram two-thirds of the way down the page there are a couple missing symbols in the two Hom groups. It should read $\text{Hom}_R(C_\ell(X; R), R) \rightarrow \text{Hom}_R(C_{k+\ell}(X; R), R)$.
- §3.3, page 251, last line. There is a missing parenthesis following the second H^j .
- §3.3, page 253. In the last paragraph of the proof of Proposition 3.42 it might be better to replace the subscripts i by k .
- §3.3, page 255, line 5. Omit the coefficient group Z . (It should have been a black-board bold \mathbb{Z} in any case.)
- §3.3, page 256, lines 1-2. Change the superscript 0 to a subscript, and change the two superscripts n to $n - 1$.
- §3.3, page 256, line 8. Change “Example 1.26” to “Example 1.24”.
- §3.3, page 258, Exercise 8, second line. Delete the second “of”.
- §3.B, page 268, tenth-to-last line. Change “homomorphism” to “bilinear map”.
- §3.B, page 272, first line. Change “for all i ” to “for all n ”
- §3.B, page 273. In the displayed equations near the bottom of the page the last $(-1)^i$ should be deleted.
- §3.B, page 276, Corollary 3B.2 (which incidentally should have been numbered 3B.8). The isomorphism in this corollary is obtained by quoting the Künneth formula

and the universal coefficient theorem, whose splittings are not natural, so the isomorphism in the corollary need not be natural as claimed. However there does exist a natural isomorphism, obtainable by applying Theorem 4.59 later in the book.

- §3.B, page 280, next-to-last line before the exercises. Change ΔT to $T\Delta$.
- §3.B, page 280, Exercise 5, lines 2 and 3. The slant products should map to the homology and cohomology of X rather than Y .
- §3.C, page 281. In the last two lines of the next-to-last paragraph, change it to read "... compact Lie groups $O(n)$, $U(n)$, and $Sp(n)$. This is explained in §3.D for $GL_n(\mathbb{R})$, and the other two cases are similar."
- §3.C, page 282, tenth line from the bottom. Change SP_{n+1} to $SP_{n+1}(X)$.
- §3.C, page 283. The summation in the displayed formula on line 14 is not sufficiently general. The formula should say

$$\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i \quad \text{where } |\alpha'_i| > 0 \text{ and } |\alpha''_i| > 0$$

There are four other places in this section where a similar correction is needed. In item (2) later on the same page it should say " $\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i$ whenever $|\alpha| > 0$, where $|\alpha'_i| > 0$ and $|\alpha''_i| > 0$." Lines 3-4 on page 284 should say "so the terms α'_i and α''_i in the coproduct formula $\Delta(\alpha) = \alpha \otimes 1 + 1 \otimes \alpha + \sum_i \alpha'_i \otimes \alpha''_i$ must be zero." On page 290, item (2), it should say " $\Delta(a) = a \otimes 1 + 1 \otimes a + \sum_i a'_i \otimes a''_i$." And in item (4) on that page it should say "the lower route gives first $\Delta(a) \otimes \Delta(b) = (\sum_i a'_i \otimes a''_i) \otimes (\sum_j b'_j \otimes b''_j)$, then after applying τ and $\pi \otimes \pi$ this becomes $\sum_{i,j} (-1)^{|a''_i||b'_j|} a'_i b'_j \otimes a''_i b''_j = (\sum_i a'_i \otimes a''_i) (\sum_j b'_j \otimes b''_j)$, which is $\Delta(a)\Delta(b)$."

- §3.C, page 286, Example 3C.5, third line. Change $2i$ to ni .
- §3.C, page 286, eleventh line up from the bottom. Modify this to say "but not in $\Gamma_{\mathbb{Z}_p}[\alpha]$ when $i > 0$, since the coproduct in $\Gamma_{\mathbb{Z}_p}[\alpha]$ is given by ..."
- §3.C, page 291, Exercise 3. Assume the H-space multiplication is associative up to homotopy.
- §3.C, page 291, Exercise 9. Add the hypothesis that X is connected.
- §3.C, page 291, Exercise 10, part (c). Assume that a_n and b_n are nonzero.
- §3.D, page 295. In the text to the left of the figure change P^n to P^{n-1} .
- §3.F, page 314, lines 9-10. The finite expressions $b_n \cdots b_1 b_0$ correspond just to nonnegative integers.
- §3.F, page 315, next-to-last line of first paragraph. Change H^n to h^n .
- §3.G, page 322, line 5. Change $H^k(X;F)$ to $H^k(\tilde{X};F)$.
- §3.G, pages 326-327. The list of Lie groups whose classifying spaces have polynomial \mathbb{Z}_p -cohomology rings is incomplete for the prime $p = 2$. Perhaps the best way to describe the situation would be to restrict the discussion to odd primes up

until the last paragraph in this section, and then enlarge the final table for the prime 2 to include the missing examples. Among these are the following Lie groups, with corresponding polynomial generators in the indicated degrees:

G_2	4, 6, 7
$Spin(7)$	4, 6, 7, 8
$Spin(8)$	4, 6, 7, 8, 8
$Spin(9)$	4, 6, 7, 8, 16
F_4	4, 6, 7, 16, 24
$PSP(2n + 1), n \geq 1$	2, 3, 8, 12, \dots , $8n + 4$

Here $PSP(n) = Sp(n)/(\pm I)$, the quotient of $Sp(n)$ by its center. I have been told there may be other examples as well, and I will post these here when I obtain a more complete list from the experts on this subject. (Note that for $p = 2$ the term ‘degree’ means the actual cohomological dimension, whereas for odd primes it meant half the cohomological dimension.)

- §3.H, page 332, line -9. Change “Bockstein” to “change-of-coefficient”.
- §3.H, page 334, line 2. Missing parenthesis in $C^n(X; E)$.
- §3.H, page 334, line following Proposition 3H.5. Repeated “the” — delete one.
- §3.H, page 335. In the statement of Theorem 3H.6, Poincaré duality with local coefficients, change the second (or alternatively, the third) occurrence of M_R to R , just ordinary coefficients in R rather than local coefficients. For more details see the separate correction page.
- §4.1, page 345, line 2. Change (X, B, x_0) to (X, A, x_0) .
- §4.1, page 359, Exercise 22. Add the word “weakly” before “homotopy equivalent”.
- §4.2, page 361, line 18. Repeated “the” — delete one of them.
- §4.2, page 370. The large diagram on this page will only commute up to sign unless the generators α are chosen carefully. Commuting up to sign is good enough for most purposes, so this isn’t really a big issue. It might be a good exercise to see how to choose generators to make the diagram commute exactly.
- §4.2, page 374. Delete the direct sum symbol \oplus at the end of the displayed exact sequence in the sixth line.
- §4.2, page 376. In the proof of injectivity of p_* there is an implicit permutation of the last two coordinates of $I^n \times I$ when the relative homotopy lifting property is applied.
- §4.2, page 380. At the end of Example 4.50 replace $K(\mathbb{Z}, 3)$ by $K(\mathbb{Z}, 4)$.
- §4.2, page 391, line 5. $H_n(X)$ should be $H_{n+1}(X)$.
- §4.3, page 398, line 3. Change SX to SA .
- §4.3, page 399, third paragraph. Change L to K' , twice.

- §4.3, page 400, line 6. Replace $h^n(\textit{point})$ by $h_n(\textit{point})$. (This typo crept in when I modified this sentence some time after the first printing, so it doesn't occur in the first printing.)
- §4.3, page 409, next-to-last line of next-to-last paragraph. Switch γ and η , so that it reads "composing the inverse path of $p\eta$ with γ ."
- §4.3, middle of page 412. In the definition of the k -invariant the coefficient group should be $\pi_{n+1}(X)$ instead of $\pi_{n+1}(K)$.
- §4.3, page 417, last line. The reference should be to Lemma 4.7 rather than to an exercise in §4.1.
- §4.3, page 419, Exercise 6. It should have been explained how the cross product is defined since we are using coefficients in G rather than a ring. However, instead of using cross products it would be better just to use Exercise 4 to construct the H-space structure and prove the stated properties. The problem could also be expanded to include showing that the H-space structure has a homotopy-inverse.
- §4.3, page 419, Exercise 8. Typo in the second line: ps should be πs .
- §4.3, page 420, Exercise 13. Small typo: It should begin "Given a map".
- §4.B, page 428, line 6. Typo: Replace Adam' by Adams'.
- §4.C, page 430, third line of Example 4C.2. Insert the word "and" before $H_{n+1}(X)$.
- §4.E, page 450. In the third-to-last line T_n should be T_{u_n} , and again in the next line as well.
- §4.F, page 454. In the last paragraph it is stated that one can associate a cohomology theory to any spectrum by setting $h^i(X) = \varinjlim \langle \Sigma^n X, K_{n+i} \rangle$. Unfortunately the wedge axiom fails with this definition. For finite wedges there is no problem, so one does get a cohomology theory for finite CW complexes. A way to avoid this problem is to associate an Ω -spectrum to a given spectrum in the way explained on the next page, then take the cohomology theory associated to this Ω -spectrum.
- §4.H, page 463. Delete the extra period at the end of the first paragraph.
- §4.H, page 464, line 14. The superscript on D should be n rather than m .
- §4.I, page 468. Exactly halfway down the page the term $J_n(X)$ should be $\Sigma J_n(X)$.
- §4.I, page 470, Exercise 2. In the first line there are three missing Σ 's. It should say $\Sigma K(\mathbb{Z}_m \times \mathbb{Z}_n, 1) \simeq \Sigma K(\mathbb{Z}_m, 1) \vee \Sigma K(\mathbb{Z}_n, 1)$. Also, in the last line the reference should be to Proposition 4I.3 instead of 4E.3.
- §4.L, page 491, seventh line from the bottom. The exponent $n + 4i$ should be $n + 2i$. The same correction should be made again on the second line of the next page.
- §4.L. Starting on page 496 and continuing for the rest of this section the name Adem is mistakenly written with an accent, as Adém. (In fact the name is pronounced with the accent on the first syllable.)