1 Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z) = x^2 + yz + xyz$.

(8 pts) (a) Find the rate of change of the potential at $P(1,1,-1)$ in the direction of the vector $\mathbf{v} = (3,4,5)$.

(4 pts) (b) In which direction does V change most rapidly at P?

(4 pts) (c) What is the maximum rate of change at P?

(18 pts) 2 Determine all the local minima, local maxima, and saddles of the function $f(x,y) = x^4 - x^2 - 2xy + y^2$.

(18 pts) 3 Using the method of Lagrange multipliers, find the points on the sphere $x^2+y^2+z^2 = 4$ that are closest and farthest from the point $(2,1,-2)$.

(18 pts) 4 Use an appropriate change of variables to evaluate

$$\iiint_R \frac{\cos\left(\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25}\right)}{\sqrt{\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25}}}\, dV$$

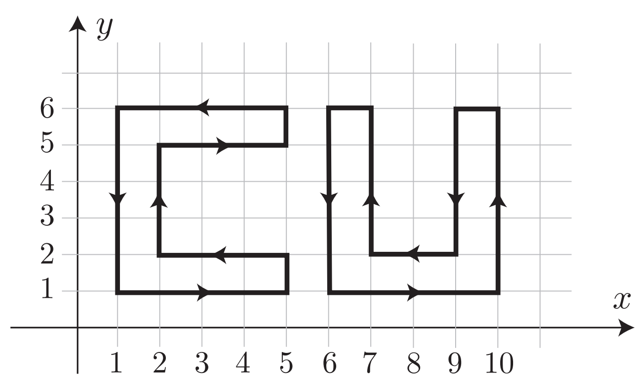
where R is the region defined by the inequalities $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \le 1$ and $\frac{z^2}{25} \le \frac{x^2}{4} + \frac{y^2}{9}$.

5 Let R be the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$, and let $\mathbf{F}(x,y,z) = (x+yz, y+xz, z+xy)$.

(9 pts) (a) Compute $\iint_{\partial R} \mathbf{F} \cdot d\mathbf{S}$.

(9 pts) (b) Determine whether the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ is path-independent. Compute this line integral for C the straight line segment from $(0,0,0)$ to $(1,1,1)$.

(18 pts) 6 Compute $\int_C 2xy\,dx + (x^2 + 2x)\,dy$ where C consists of the two closed curves shown in the figure at the right.



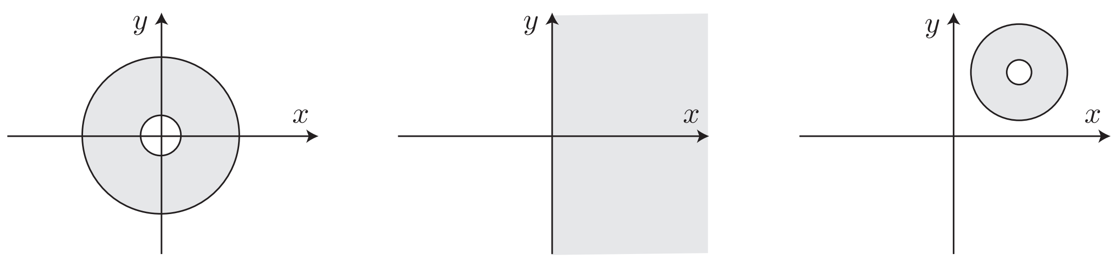
More problems on the back!

7 Let $\mathbf{F}(x, y)$ be the vector field $\left(\dfrac{x-y}{x^2+y^2}, \dfrac{x+y}{x^2+y^2}\right)$.

(6 pts) (a) Compute curl($\mathbf{F}$).

(6 pts) (b) Compute the line integral of $\mathbf{F}$ around the circle $x^2 + y^2 = a^2$.

(6 pts) (c) In which of the following regions is $\mathbf{F}$ the gradient field of some function $f(x, y)$ defined throughout the region? Give reasons for your answers. (The middle region is the points (x, y) with $x > 0$.)



(6 pts) 8 (a) Compute $\displaystyle\int_{\partial S} xy^2\,dx + x\,dy + z^2\,dz$ where S is the portion of the cylinder $x^2+y^2 = 1$ lying between the graphs $z = f(x, y)$ and $z = g(x, y)$ for arbitrary smooth functions f and g such that $f(x, y) < g(x, y)$ for all (x, y). (Use the orientation for ∂S induced from the outward normal orientation for S.)

(6 pts) (b) Using your answer in part (a), show that the line integral $\displaystyle\int_C xy^2\,dx + x\,dy + z^2\,dz$ takes the same value for all closed curves C formed by the intersection of the cylinder with a graph $z = f(x, y)$. (Use the orientation of C that is counterclockwise, viewed from above.)

(6 pts) (c) Compute the value of the line integral in (b) using a convenient choice of the curve C.

(18 pts) 9 Find the surface area of the boundary surface ∂R of the solid region R consisting of all points (x, y, z) satisfying $x^2 + z^2 \leq 1$ and $y^2 + z^2 \leq 1$.

Now relax and enjoy your summer!