Teaching Statement — Christopher Hardin

> For me, this was among all the marvels that I discovered in nature the most marvelous of all, and I must say, that for my part, no more pleasant sight has met my eye than this of so many thousand living creatures in one small drop of water.
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> —Antoni van Leeuwenhoek, on his discovery of bacteria

Many students go through high school feeling that their classes are just something to endure for the sake of acquiring knowledge that will be useful later. Their environment reinforces this: “impractical” subjects such as music and art are expendable, college is promoted only as a career enhancer, and classes are taken for the sake of getting into college, not for fun. The message is that education is merely a vehicle, not a destination. Mathematics especially receives this treatment, even from within; posters declare that “Math is power!” or “Math opens doors” (admittedly, “Math is a worthwhile pursuit unto itself” is less catchy). While these are not lies, they discourage a direct appreciation of math by only emphasizing its value as a tool.

While embracing its role as a vehicle, colleges have the opportunity to show that an education can also be much more than that, by creating an environment where students find intellectual excitement and personal fulfillment in their studies. Amherst provided me with such an environment as an undergraduate; subjects that had seemed boring and sterile in high school were suddenly full of life. While it was a gradual transformation, this immersion in academic richness and consistent mental challenge deeply changed me; I left Amherst with an intellectual ardor that sustains and motivates me, and a sense that I can significantly contribute to the world if I involve myself. Not everyone is changed in the same way, but most Amherst alumni would agree that their undergraduate experience had a profound impact on their lives, well beyond just professional preparation.

My goal in teaching is to contribute to this kind of experience. In other words, I wish to succeed at teaching students mathematics, and beyond that, to do so in a way that fosters general intellectual enthusiasm, inside or outside mathematics.

This goal is hardest to achieve in introductory calculus courses, where many of the students only enroll to fulfill a requirement. For those students who truly won’t need calculus for their major (or medical school), I think courses that focus on developing the ability to form rigorous arguments would do much more for them. But ultimately, there are going to be many students taking calculus who would rather not. While I don’t fool myself into believing I will convert confessed math-haters into math majors, I can still get through to them and exceed their expectations about what their calculus experience will be. I feel that the foundation is commitment, clarity, patience, and enthusiasm.

I begin by learning the students’ names as soon as possible. I am usually able to name all my students by the end of the second meeting I have with them. *They really like that.* I also poll them to pick office hours that work best for them, and encourage them to email me and make an appointment if they still cannot come to my office hours, or if they do not feel they are getting enough individual attention during my office hours.

When choosing examples for non-math majors, I find that it helps when they have some kind of aesthetic appeal outside of mathematics. One that students react well to is calculating the volume of the Gehry Tower in Hanover, Germany. The building, roughly, is a rectangular solid that has been twisted along its vertical axis. We set up the integral for its volume, evaluate it, and compare it to the known volume of the building; this conveys relevance to the students. I ask the students what simpler shape would have given us the same integral. *A rectangular solid.* Then, I ask the students how we could have calculated the volume without integration. *Find the volume of the rectangular solid.* The students have now uncovered Cavalieri's Principle. They get a sense of both elegance and accomplishment—the problem, which they would have found unapproachable a day earlier, became first doable, and then outright trivial when looked at correctly. But what gets them engaged in the first place is that they like the architecture.

When time allows, I often break students into groups of three or four to work on group exercises together for the last 10–15 minutes of class. I prepare worksheets that generally start with a straightforward application of the material from class, as a warm up, and then lead to more subtle questions that require thought and discussion. This serves a number of purposes; in particular: immediate application of material from lecture helps the students understand and remember it better; forcing students to verbalize mathematics sharpens their understanding of it; questions that do not arise during lecture often arise when students first try to use the material, and group work brings out those questions while I am there—and when possible, other students answer them.

Many educational uses of computers seem contrived to me, but when properly applied, computers can greatly assist students in learning mathematics. I have spent a great deal of time developing a graphing program for functions of 2 variables, because I could not find any existing software that did what I wanted. After I work a problem on the board algebraically, such as finding a tangent plane or finding maxima and minima, I like to revisit the problem in my program (projected onto a screen), and go through the same steps visually; after graphing the function, I can display tangent planes, display contour lines, display the curve where $f_x = 0$ and the curve where $f_y = 0$ (so that the these curves intersect at critical points), or colour the surface according to the discriminant or Laplacian. I can also use constants in the formula, and dynamically alter them to illustrate their effect on the graph. Although I currently only use the program in class, my goal is to document it and make it available to my students and other instructors.

For courses beyond calculus, I think the focus should be on developing students' ability to prove theorems, even if it comes at the expense of how much material the students get to see. In particular, homework assignments should primarily be proof-oriented. (A partial exception to this is complex analysis, in which applying known theorems can still require great ingenuity on the student's part, as when finding appropriate paths for evaluating integrals. *I.e.*, the calculations take on the flavor of proofs.) Having had extremely positive experiences with the Moore Method as an undergraduate, I recommend its use in at least some upper-level courses. Undergraduate research obviously holds the greatest potential rewards for students, and should be encouraged; the challenge for faculty is helping students find topics that are interesting, accessible, and not already completely understood. Because old topics generally fail at least one of these criteria, it is important for faculty to remain active within their fields, as I intend to do.

Ideally, I see the college experience as being to the student what the discovery of bacteria was to Leeuwenhoek. The faculty, as microscope, help the student to see that life is awash in elegant concepts and intellectual reward for those who are willing to make the effort to look closely. Superb teaching inspires this effort.