BASIC SURVIVAL TOOLS FOR MATHEMATICS.

ERIN P. J. PEARSE

These problems are from Munkres, Chapter 1.3. You will probably find a clear understanding of these principles to be helpful in any mathematics course you take, from now on. The best way to remember and understand them is to **prove** them!

1. Let $f : X \to Y$. Let $U \subseteq X$ and $V \subseteq Y$.
   (a) Show that $U \subseteq f^{-1}(f(U))$. Show that equality holds if f is injective. Find an example of when U is a proper subset of $f^{-1}(f(U))$ (i.e., when $U \neq f^{-1}(f(U))$.
   (b) Show that $f(f^{-1}(V)) \subseteq V$. Show that equality holds if f is surjective. Find an example of when $f(f^{-1}(V))$ is a proper subset of V.

   Key Point: you *cannot* "cancel" f by applying f^{-1}, unless you know f is injective. Similarly, you *cannot* "cancel" f^{-1} by applying f, unless you know f is surjective.

2. Let $f : X \to Y$ and let $V_i \subseteq Y$.
   Show that f^{-1} preserves inclusions, unions, intersections, and differences (hence also complements) of sets:
   (a) $V_0 \subseteq V_1 \implies f^{-1}(V_0) \subseteq f^{-1}(V_1)$.
   (b) $f^{-1}(V_0 \cup V_1) = f^{-1}(V_0) \cup f^{-1}(V_1)$.
   (c) $f^{-1}(V_0 \cap V_1) = f^{-1}(V_0) \cap f^{-1}(V_1)$.
   (d) $f^{-1}(V_0 \sim V_1) = f^{-1}(V_0) \sim f^{-1}(V_1)$. (Hence $f^{-1}(V_0^C) = (f^{-1}(V_0))^C$.)

3. Let $f : X \to Y$ and let $U_i \subseteq X$.
   Show that f preserves inclusions, and unions, but does not necessarily preserve intersections or differences of sets:
   (a) $U_0 \subseteq U_1 \implies f(U_0) \subseteq f(U_1)$.
   (b) $f(U_0 \cup U_1) = f(U_0) \cup f(U_1)$.
   (c) $f(U_0 \cap U_1) \subseteq f(U_0) \cap f(U_1)$. Show that equality holds if f is injective. Find an example of when $f(U_0 \cap U_1)$ is a proper subset of $f(U_0) \cap f(U_1)$.
   (d) $f(U_0 \sim U_1) \supseteq f(U_0) \sim f(U_1)$. Show that equality holds if f is injective. Find an example of when $f(U_0) \sim f(U_1)$ is a proper subset of $f(U_0 \sim U_1)$.

I also recommend reading over problems 4 & 5 from this section (p. 21).

April 17, 2005.