Math 4550 HW due Mar. 27, 2009

1. Let $n, m \in (\mathbb{N}, |)$. Determine $\mu(n, m)$ for all possible n, m.
2. A *partition* of $[n]$ is a set of subsets $A_1, \dots, A_m$ such that $A_i \cap A_j = \emptyset$ for all $i \neq j$ and $\cup_{i=1}^m A_i = [n]$. The set of all partitions of $[n]$ is denoted Π_n. It has a poset structure given by refinement. A partition $\sigma_1 = \{A_1, \dots, A_m\}$ of $[n]$ is a refinement of another partition $\sigma_2 = \{B_1, \dots, B_k\}$ of $[n]$ if every A_i is a subset of some B_j. For instance, $\{\{1,2\}, \{3,5\}, \{4\}\}$ is a refinement of $\{\{1,2,4\}, \{3,5\}\}$. The poset structure of Π_n is given by $\sigma_1 \leq \sigma_2$ if σ_1 is a refinement of σ_2. You can see that the least element, which we denote by $\hat{0}$, of Π_n is $\{\{1\}, \{2\}, \dots, \{n\}\}$ and the greatest element, which we denote by $\hat{1}$, is $\{1, 2, \dots, n\}$. Compute $\mu_{\Pi_n}(\hat{0}, \hat{1})$ for $n = 2, 3$ and 4.
3. Let $K \subseteq \mathbb{R}^d$. Assume that $\vec{0} \in K$. Show that the affine span of K is equal to the linear span of K. Prove that the dimension of the linear span (= affine span) of K is strictly less than d if and only if K^Δ contains a line through the origin.