Math 4550 HW due Mar. 5, 2009

1. Prove that there is no simplicial 5-polytope whose f-vector (beginning with f_0) is $(7, 22, 38, 35, 14, 1)$.

2. (The Upper Bound Theorem) Let P be a simplicial d-polytope with n vertices. Prove that for all $i, f_i(P) \leq f_i(C(n, d))$. (You may use the fact that if $a < b$, then $a^{<i>} < b^{<i>}$.)

3. For a polytope P let $\mathcal{F}(P)$ be the set of faces of P. Construct a bijection ϕ from $\mathcal{F}(\square^d)$ to $\mathcal{F}(\diamond^d)$ such that if G and H are faces of $\square^d$ with $G \subseteq H$, then $\phi(G) \supseteq \phi(H)$.