

Math 4550 HW due Feb. 26, 2009

1. Let  $\Delta$  be the abstract simplicial complex whose vertices are  $V = \{1, 2, 3, 4, 5, 6\}$  and whose facets are

$$\{1, 4, 5\}, \{2, 3, 6\}, \{3, 5, 6\}, \{1, 2, 4\}, \{1, 3, 5\}, \{2, 4, 6\}.$$

Prove that  $\Delta$  is not shellable.

2. Let  $\Delta$  be an abstract simplicial complex with vertex set  $V$ . The *cone* of  $\Delta$ , denoted  $C(\Delta)$ , is the abstract simplicial complex whose vertex set is  $V \cup \{x\}$ , where  $x$  is any new vertex not in  $V$ , and whose facets are all subsets of the form  $F \cup \{x\}$ , where  $F$  is a facet of  $\Delta$ . Show that  $f_{C(\Delta)}(t) = (t + 1)f_{\Delta}(t)$  and  $h_{C(\Delta)}(t) = t h_{\Delta}(t)$ .
3. Let  $\Delta$  be the boundary of  $\Delta^d$ . So  $\Delta$  is a  $(d-1)$ -dimensional simplicial complex. Prove that *any* ordering of the facets of  $\Delta$  is a shelling of  $\Delta$ . Determine  $h_{\Delta}(t)$ .