Math 4550 HW due Feb. 12, 2009

1. A polytope P is *symmetric* if $x \in P$ if and only if $-x \in P$. Prove that if P is a symmetric $\mathcal{V}$-polytope in $\mathbb{R}^d$, then there exists some (possibly many) e and an affine map $T : \mathbb{R}^e \to \mathbb{R}^d$ such that $T(\diamond^e) = P$.

2. The d-permutahedron is the convex hull of all permutations of the vector $\begin{bmatrix} 1 \\ 2 \\ \vdots \\ d \end{bmatrix}$. For instance, the 3-permutahedron is the convex hull of

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

   You can check that this is a hexagon in $\mathbb{R}^3$.

   (a) Prove that the dimension of the d-permutaheron is $d - 1$.

   (b) Prove that all of the $d!$ permutations are extreme points of the d-permutahedron.

3. Give an example of a closed convex set K, a face F of K and a face G of F where G is NOT a face of K.