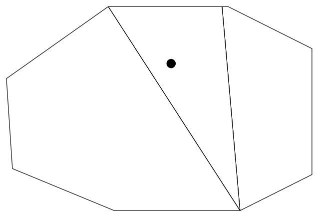

Figure 1:

Math 4550 HW2 due Feb. 5, 2009

1. Show that the number of faces of each dimension of $C(6,3)$ is the same as the octahedron. Are the octahedron and $C(6,3)$ combinatorially equivalent?

2. Prove, by using the definition, that every vertex of $\square^d$ is a zero-dimensional face of $\square^d$.

3. Let $X = \{x_1, \ldots, x_n\}$ and $Y = \{y_1, \ldots, y_n, y_{n+1}\}$ be finite subsets of $\mathbb{R}^d$. The two sets X and Y may, or may not, have elements in common. Prove that if X is an affinely independent set, and Y is an affinely independent set, then there exists at least one y_j such that $X \cup \{y_j\}$ is affinely independent.

4. Let $X = \{x_1, \ldots, x_n\}$ be a finite subset of $\mathbb{R}^d$. Last week we saw that every element y of the convex hull of X can be written as a convex combination $c_1x_1 + \cdots + c_kx_k$ of elements of X. Prove that this can be done with $k \leq d+1$. In other words, y is in the convex hull a subset of X consisting of at most $d+1$ points. For instance, in an octagon, which is the convex hull of 8 points in $\mathbb{R}^2$, every point is actually in some triangle whose vertices are the vertices of the octagon. (See figure 1)