1. Show that any acyclic orientation of a (finite) graph has at least one source and one sink.

2. Find polytopes $P$ and $Q$ such that $G(P)$ is isomorphic to $G(Q)$ and $\dim P \neq \dim Q$.

3. A different approach to the chromatic polynomial is through deletion-contraction. Let $G$ be a graph. It may have parallel edges (more than one edge between a pair of vertices) and/or loops. Define $\chi_G(\lambda)$ as before. If $G$ has a loop, then $\chi(G) \equiv 0$. Let $e$ be an edge of $G$. The deletion of $e$, denoted $G - e$, is the graph obtained by removing the edge $e$. The contraction of $G$ along $e$, denoted $G/e$ is the graph obtained by contracting the edge down to a vertex and identifying the two vertices of the edge down to one vertex. This may introduce loops and/or parallel edges. For instance, if $G$ is a triangle, then the contraction of $G$ along any of its edges is a graph with two vertices and two parallel edges. Prove that for any edge $e$ of a loopless graph,

$$\chi_G(\lambda) = \chi_{G-e}(\lambda) - \chi_{G/e}(\lambda).$$

Use this to show that the signs of the coefficients of $\chi_G(\lambda)$ alternate in sign.