

**Math 4550 HW1 due Jan. 29, 2009**

1. (a) Prove that a closed half-space is convex.  
(b) Give an example of a nonempty convex set  $A \subseteq \mathbb{R}^d$  and  $x \notin A$  such that there is no point in  $A$  closest to  $x$ . Specifically, for all  $y \in A$  there exists  $z \in A$  such that  $\|x - z\| < \|x - y\|$ .
2. Let  $T : \mathbb{R}^d \rightarrow \mathbb{R}^e$  be an affine transformation and  $A$  an affine subspace of  $\mathbb{R}^d$ . Prove that

$$T(A) = \{y \in \mathbb{R}^e : y = T(x), x \in A\}$$

is an affine subspace of  $\mathbb{R}^e$ .

3. Let  $P$  be a  $\mathcal{V}$ -polytope in  $\mathbb{R}^d$ . Prove that for some  $e$  there exists an affine map  $T : \mathbb{R}^{e+1} \rightarrow \mathbb{R}^d$  such that  $T(\Delta^e) = P$ .