

**Math 4550 Questions for March 24, 2009**

1. Find as many (combinatorially) self-dual polytopes as you can.
2. A polytope is *simple* if it is dual to a simplicial polytope. Prove that a  $d$ -polytope is simple if and only if every vertex is contained in exactly  $d$  edges. The  $d$ -simplex is both simple and simplicial. What, if any, other polytopes are both simple and simplicial?
3. Compute  $\mu_{B_n}(\emptyset, A)$  for any  $A \subseteq [n]$ . Use this to compute  $\mu_{B_n}(A, B)$  for any  $A \subseteq B \subseteq [n]$ .
4. Use the previous question to compute  $\mu_{\mathbb{N}}(1, n)$  for  $n$  when  $n$  is a product of distinct primes. What is  $\mu_{\mathbb{N}}(1, n)$  when  $n$  is not a product of distinct primes? (Here,  $\mathbb{N}$  is the poset of natural numbers with  $a \leq b$  iff  $a$  divides  $b$ .)
5. Deduce by trying examples the value of  $\mu_{\mathcal{F}(P)}(\emptyset, P)$  for  $P$  a  $d$ -polytope. Prove your answer.