1. Let $G \subseteq F$ be two faces of a polytope. Prove that there exists a polytope $Q$ such that the interval $[G, F]$ in $\mathcal{F}(P)$ is isomorphic to $\mathcal{F}(Q)$.

2. Let $P$ be a polytope. Prove that $\mathcal{F}(P)$ is graded and that for any face $F$ of $P$ the rank of $F$ is equal to $\dim F + 1$.

3. Prove that if $P$ is a polytope, then $\mathcal{F}(P)$ is an Eulerian poset.

4. Problem 45 and Cor. 6.1.1 of the ‘text’.

5. Establish the Möbius inversion formulas.