

Math 4550 Questions for March 10, 2009

1. Prove that the  $h$ -vector of a the boundary of a  $d$ -dimensional stacked polytope with  $n$  vertices is  $(1, \underbrace{n-d, \dots, n-d}_{d-2 \text{ times}}, 1)$ .
2. For what values of  $n$  is the interval  $[1, n]$  in the poset  $(\mathbb{N}, |)$  isomorphic to  $B_m$  for some  $m$ ?
3. Let  $F$  and  $G$  be faces of a polytope  $P$ . Prove that  $F \cup G$  is a face of  $P$ .
4. Let  $F$  be a face of a poytope  $P$ . Let  $G$  be a face of  $F$ . Prove that  $G$  is a face of  $P$ .
5. If  $K \subseteq K'$ , then  $K^\Delta \supseteq (K')^\Delta$ . Identify  $\mathbb{R}^d$  with  $((\mathbb{R}^d)^*)^*$ . Then  $K^{\Delta\Delta} \supseteq K$ . Give an example to show that this inclusion may be strict. Prove that if  $K$  is a  $d$ -polytope with the origin in its interior, then  $K^{\Delta\Delta} = K$ .