

**Math 4550 Questions for March 3, 2009**

1. Let  $\Delta$  be the boundary of a  $d$ -polytope. Prove that every face of dimension  $d-2$  is contained in exactly two facets of  $\Delta$ .
2. Compute the  $h$ -vector of the boundary of  $C(n, d)$  (the cyclic polytope of dimension  $d$  with  $n$  vertices).
3. Compute the  $h$ -vector of the  $d$ -dimensional cross polytope. (Even better - find a shelling that 'explains' your result.)
4. Let  $P$  be a  $d$ -polytope. Prove Euler's formula

$$\sum_{i=0}^d (-1)^i f_i(P) = 1.$$

5. Let  $a, i$  be positive integers. Write

$$a = \binom{a_i}{i} + \binom{a_{i-1}}{i-1} + \cdots + \binom{a_j}{j},$$

where at each step, beginning with  $a_i$ , you choose  $a_k$  to be as large as possible. Show that

$$a_i > a_{i-1} > \cdots > a_j.$$