Math 4550 Questions for March 8, 2011

1. Prove that Stanley’s trick works. What is the $h$ to $f$ version of Stanley’s trick?

2. Let $G$ be a simple graph with $n$ edges and $v$ vertices. Let $\Delta$ be the abstract simplicial complex whose vertices are the edges of $G$ and whose faces are those subsets of edges whose removal does NOT disconnect the graph. Now assume that each edge has independent probability $p$, $0 < p < 1$ of being removed. Prove that the probability that the graph remains connected is

\[(1 - p)^{v-1}[h_0 + h_1p + \cdots + h_{n-v+1}p^{n-v+1}],\]

where $(h_0, \ldots, h_{n-v+1})$ is the $h$-vector of $\Delta$. (You may assume that $\Delta$ is shellable. Also, you may assume that any connected graph with $v$ vertices has at least $v - 1$ edges and that any subgraph of $G$ which contains less than $v - 1$ edges is disconnected.)

3. Problem 30 of the ‘text’.

4. Let $G$ be a connected graph. Let $\Delta$ be the abstract simplicial complex whose vertices are the vertices of $G$ and whose faces are the empty set, the vertices of $G$ and all pairs $\{p, q\}$ when $\{p, q\}$ is an edge of $G$. Prove that $G$ is shellable.