

Math 4550 Questions for Feb. 24, 2009

1. Let Δ be a d -dimensional abstract simplicial complex. Prove there exists a geometric simplicial complex in \mathbb{R}^{2d+1} whose associated abstract simplicial complex is Δ .
2. Suppose that Δ' is a $(d-1)$ -dimensional simplicial complex obtained from Δ by an i -shelling step. Prove that $f_{\Delta'}(t) = f_{\Delta}(t) + (t+1)^{d-i}$.
3. Find a formula for f_i in terms of h_0, \dots, h_{i+1} . Find a formula for h_i in terms of $f_{-1}, f_0, \dots, f_{i-1}$.
4. Let Δ be the boundary of a simplicial 3-polytope. Prove that $h_0 = h_3 = 1$ and $h_1 = h_2$.
5. Prove that Stanley's trick computes h_i correctly.