Math 4550 Questions for Feb. 17, 2009

1. Prove that a $\mathcal{V}$-polytope is a closed set.

2. Let K be a convex set and $[a, b]$ a line segment in K with a in the interior of K. Prove that for all c in the line segment, except possibly b, c is in the interior of K.

3. Let K be a closed convex set with $\vec{0}$ in the interior of K. For $r \in \mathbb{R}$ define $rK = \{ry : y \in K.\}$ If x is on the boundary of K, then for all $0 < \varepsilon < 1,\ x$ is not in $(1 - \varepsilon)K$, so by previous results the hyperplane H_ε which is the bisector of a line segment between x and the unique point of K closest to x separates x and K. Make as rigorous as you can the idea that the limit of the H_e hyperplanes is a hyperplane H such that x is on the proper face of K equal to $H \cap K$.

4. Let P be an $\mathcal{H}$-polytope. Write $P = \bigcap_{i=1}^{m} H_i$ minimally. Specifically, for all $1 \leq j \leq m, P \neq \bigcap_{i \neq j} H_j$. Prove that the boundary of P is contained in the union of the m corresponding to the $H_i \cap P$.

5. Derive formulas for f_1 and f_2 in terms of f_0 for a simplicial 3-polytope. (You may assume Euler's formula $f_2 - f_1 + f_0 = 2$.)