

Math 4550 Questions for Feb. 10, 2009

1. Let K be a closed convex subset of \mathbb{R}^d and x a point in \mathbb{R}^d not in K . Prove that there is a *unique* point $y \in K$ closest to x .
2. Let K and K' be closed convex subsets of \mathbb{R}^d with K bounded.

(a) Prove there exists $x \in K$ and $y \in K'$ such that for any $x' \in K$ and $y' \in K'$

$$\|x - y\| \leq \|x' - y'\|.$$

- (b) Prove that if $K \cap K' = \emptyset$, then there exists a separating hyperplane for K and K' . (Note: A separating hyperplane does NOT contain any points of K or K' .)
- (c) If the hypothesis that K is bounded is removed, does the conclusion of the previous problem still hold?
3. Prove that a vertex (0-dimensional face) of a convex set is an extreme point of the set.
 4. Suppose that K is the convex hull of $A \subseteq \mathbb{R}^d$. Prove that the extreme points of K are a subset of A .
 5. Prove that if F is a face of a convex set K , then the extreme points of F equal the intersection of F and the extreme points of K .