

Math 4550 Questions for Feb. 3, 2009

1. What are the faces of \square^d ? What are the faces of \diamond^d ? How many faces are there in each dimension for \square^d and \diamond^d ? How would you show that every subset of vertices of Δ^d form a face?
2. Prove that the convex hull of $d + 1$ affinely independent points is affinely equivalent to Δ^d .
3. Prove that if $t_{i_1} < t_{i_2} < \dots < t_{i_{d+1}}$, then the matrix

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ t_{i_1} & t_{i_2} & \dots & t_{i_{d+1}} \\ \vdots & \vdots & & \vdots \\ t_{i_1}^d & t_{i_2}^d & \dots & t_{i_{d+1}}^d \end{bmatrix}$$

is invertible. Better, prove that the determinant is

$$\prod_{j>k} (t_{i_j} - t_{i_k}).$$

4. Let F be a subset of the vertices of $C(n, d)$ such that $|F| \leq d/2$. Prove that the vertices of F are the vertices of a face of $C(n, d)$.