

Math 4550 Questions for Jan 27, 2009

1. Let x and y be two points in a nonempty affine subspace A . Does $A - x = A - y$?
2. Let $X \subseteq \mathbb{R}^d$. Show that the smallest affine subspace containing X equals the set of all possible affine combinations of elements of X .
3. Prove that if A is a nonempty affine subspace, then $\dim A$ is the maximum number of affinely independent elements in A minus one.
4. Let $X = \{x_1, \dots, x_n\}$ be a subset of \mathbb{R}^d . Prove that X is affinely independent if and only if the set

$$\left\{ \begin{bmatrix} 1 \\ x_1 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ x_n \end{bmatrix} \right\}$$

is linearly independent in \mathbb{R}^{d+1} .

5. Describe Δ^d as an \mathcal{H} -polytope.
6. Prove that if $X \subseteq \mathbb{R}^d$, then the convex hull of X consists of all convex combinations of elements of X .
7. Let $T : \mathbb{R}^d \rightarrow \mathbb{R}^e$ be an affine transformation. Does there always exist a linear transformation $L : \mathbb{R}^d \rightarrow \mathbb{R}^e$ and an element $v \in \mathbb{R}^d$ such that $T(x) = L(x + v)$? If so, prove it. If not, give a criterion to determine when such an L and v do exist.