
1. Let $A \subseteq \mathbb{R}^d$ be an affine subspace, $A = W + v = W' + v'$, $W$ and $W'$ linear subspaces of $\mathbb{R}^d$. Prove that $W = W'$. Show that for any $y \in A$, $A + (-y) = W$.

2. Let $A \subseteq \mathbb{R}^d$. Show that the affine span of $A$ equals the set of all possible affine combinations of elements of $A$.

3. Let $H$ be a linear subspace of $\mathbb{R}^d$ of dimension $d - 1$. If $A$ is a linear subspace of $\mathbb{R}^d$ and $A \not\subseteq H$, then $\dim A \cap H = \dim A - 1$.

4. Prove that $A = \{x_1, \ldots, x_n\} \subseteq \mathbb{R}^d$ is affinely independent if and only if no $x_i$ is an affine combination of the other elements of $A$.

5. Let $K$ be a convex subset of $\mathbb{R}^d$ and $f : \mathbb{R}^d \to \mathbb{R}^e$ be an affine map. Is $f(K)$ convex? If $K$ is a convex subset of $\mathbb{R}^e$ is $f^{-1}(K)$ a convex subset of $\mathbb{R}^d$?

6. Prove that if $A \subseteq \mathbb{R}^d$, then the convex hull of $A$ consists of all convex combinations of elements of $A$.

7. Let $A \subseteq \mathbb{R}^d$ and $y \in ch(A)$. Prove that there exists $x_1, \ldots, x_{d+1}$ such that $y \in ch(x_1, \ldots, x_{d+1})$.

8. Let $\Delta$ be a $d$-simplex in $\mathbb{R}^e$. Prove that there exists an affine map $f : \mathbb{R}^{d+1} \to \mathbb{R}^e$ such that $f(\Delta^d) = \Delta$ and $f$ is a bijection when restricted to $\Delta^d$. 