Math 4320 HW6 due April 17, 2013

1. Problem 3.20, pg. 235 in the text.

2. Problem 3.32, pg. 242 of the text.

3. Problem 3.33, pg. 242 of the text.

4. Problem 3.53, pg. 252 of the text.

5. Problem 3.54, pg. 252 Note: In class we showed that if \((m, n) = 1\), then the function \(f : \mathbb{I}_{mn} \to \mathbb{I}_m \times \mathbb{I}_n\) given by \([j]_{mn} \to ([j]_m, [j]_n)\), where the subscripts tell what group the coset is in, is a group isomorphism. So you may assume this.

6. Let \(R\) be the subring of \(\mathbb{C}\) generated by the integers and \(\sqrt{-5}\).
   
   (a) Prove that \(R = \{a + b\sqrt{5} \, i \in \mathbb{C} : a, b \in \mathbb{Z}\}\).
   
   (b) For \(u = a + b\sqrt{5} \, i \in R\) define \(N(u) = a^2 + 5b^2\) in \(\mathbb{Z}\). Prove \(N(u \cdot v) = N(u) \cdot N(v)\), if \(u\) is a unit in \(R\), then \(N(u) = 1\). Conclude that the only units of \(R\) are \(\pm 1\).
   
   (c) Prove that 2, 3, 1 + \(\sqrt{5}\) \(i\), and 1 \(−\sqrt{5}\) \(i\) are irreducible in \(R\).

Remark: Notice that in this ring 6 = 2 \(\cdot\) 3 = (1 + \(\sqrt{5}\) \(i\)) \(\cdot\) (1 \(−\sqrt{5}\) \(i\)). So unique factorization into irreducibles does not hold in this ring.