You can find here the solutions to the problems whose solution is not on the back of the book.

Problem 2.2.5

Solution:
$6x + 4y$ is 2 times $3x + 2y$. There is no solution unless the right side is $2 \cdot 10 = 20$. Then all the points on the line $3x + 2y = 10$ are solutions, including $(0, 5)$ and $(4, -1)$. (The two lines in the row picture are the same line, containing all solutions).

Problem 2.2.12

Solution:
Elimination leads to an upper triangular system; then comes back substitution. So, we have the system
\[
\begin{bmatrix}
2 & 3 & 1 \\
0 & 1 & 3 \\
0 & 0 & 8
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
=
\begin{bmatrix}
8 \\
4 \\
8
\end{bmatrix}.
\]

The third equation gives $z = 1$, which then means that, plugging in this value for $z$ in the second equation, $y = 1$. Finally, using these values for $y$ and $z$ in the first equation, $x = 2$. Thus we get $(x, y, z) = (2, 1, 1)$.

Problem 2.3.8

Solution:
Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, $M^* = \begin{bmatrix} a & -\ell b \\ c-\ell a & d-\ell b \end{bmatrix}$. Hence, we have that $\det M^* = a(d-\ell b) - b(c-\ell a) = ad - a\ell b - bc + a\ell b$. Thus, $\det M^* = ad - bc$.

Problem 2.3.13

Solution:

1. $E$ times the third column of $B$ is the third column of $EB$. Thus a column that stars at zero will stay at zero.

2. $E$ could add row 2 to row 3 to change a zero row to a nonzero row.
Additional Problem

Solution:
A lot of people failed to solve this problem. Everyone should read this solution carefully. Let \( f(x) = \frac{a}{2} + bx + cx^2 \). Then, \( f(1) = 2 \) gives us the equation \( f(1) = a + b + c = 2 \). Similarly, \( f(-1) = -2 \) gives us \( f(-1) = -a - b + c = -2 \). Finally, \( f(2) = 9.5 \) gives us \( f(2) = \frac{a}{2} + 2b + 4c = \frac{19}{2} \). Hence, the matrix equation becomes:

\[
\begin{bmatrix}
1 & 1 & 1 \\
-1 & -1 & 1 \\
\frac{1}{2} & 2 & 4
\end{bmatrix}
\begin{bmatrix}
a \\ b \\ c
\end{bmatrix}
= 
\begin{bmatrix}
2 \\ -2 \\ \frac{19}{2}
\end{bmatrix}.
\]

Adding the first row to the second row, and substituting the second row for this, we get:

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 0 & 2 \\
\frac{1}{2} & 2 & 4
\end{bmatrix}
\begin{bmatrix}
a \\ b \\ c
\end{bmatrix}
= 
\begin{bmatrix}
2 \\ 0 \\ \frac{19}{2}
\end{bmatrix}.
\]

This means that \( c = 0 \). Finally, substituting the first equation by the first row minus twice the third row, we get:

\[
\begin{bmatrix}
0 & -3 & -7 \\
0 & 0 & 2 \\
\frac{1}{2} & 2 & 4
\end{bmatrix}
\begin{bmatrix}
a \\ b \\ c
\end{bmatrix}
= 
\begin{bmatrix}
-17 \\ 0 \\ \frac{19}{2}
\end{bmatrix}.
\]

The first equation is \(-3b - 7c = -17\), but because \( c = 0 \), we get \( b = \frac{17}{3} \). Thus, from the first equation, we get \( a = -\frac{11}{3} \). The function becomes \( f(x) = -\frac{11}{3}x + \frac{17x}{3} \).

Problem 2.4.6

Solution:
If you got this problem wrong, you need to go over matrix multiplication and addition! Now, on the one hand \((A + B)^2 = \begin{bmatrix} \frac{10}{3} & 4 \\ 6 & \frac{6}{3} \end{bmatrix} = A^2 + AB + BA + B^2 \). But on the other hand \( A^2 + 2AB + B^2 = \begin{bmatrix} \frac{16}{3} & 2 \\ \frac{3}{3} & \frac{6}{3} \end{bmatrix} \). Hence, in this case, \((A + B)^2 \neq A^2 + 2AB + B^2 \).

Problem 2.4.14

1. True (\( A^2 \) is only defined when \( A \) is square).

2. False (is \( A \) is an \( m \) by \( n \) matrix and \( B \) is an \( n \) by \( m \) matrix, then \( AB \) is and \( m \) by \( m \) matrix and \( BA \) is an \( n \) by \( n \) matrix).

3. True (for the same reasons as above).

4. False (take \( B = 0 \). This case comes up constantly, so do not dismiss it).
Problem 2.4.26

Solution:
Columns of $A$ times rows of $B$

\[
\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 6 & 6 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 8 & 4 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1 \end{bmatrix} = AB.
\]