Early warning signals of abrupt changes in ecosystems

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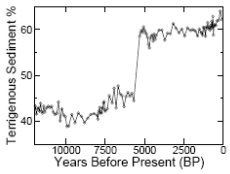
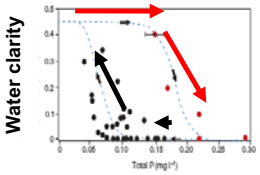
Motivation

- Ecosystems can occasionally exhibit abrupt shifts as a response to smoothly changing external conditions.
- They are known as "Regime shifts".
  - Desertification
  - Lakes
- They can be:
  - Abrupt
  - Hysteretic
- Can lead to loss of vital services they provide to humans



Example from Sahara



Lakes in Netherlands

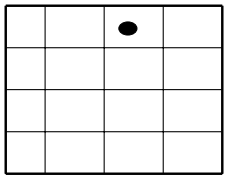
P. deMenocal et al. / Quaternary Science Reviews 19 (2000) 347–361

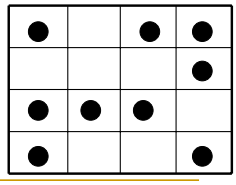
Meijer, 2000, Biomonipulation in the Netherlands; Scheffer et al, Nature, 2001, 413: 591-596

Ecological question

- Are we approaching the vicinity of a threshold?
  - Can we identify "**Universal features**"?
  - That can be applied across several ecological systems?
- Given *time series data* of a state variable at one site:
  - How do we assess proximity to a transition?
  - Quick review of other indicators
  - ***Changing skewness:*** an early warning signal
- Given time series of full/partial *spatial data:*
  - Patches, ***Spatial variance, Spatial skewness*** can provide early warning signals.

Ecosystem in space





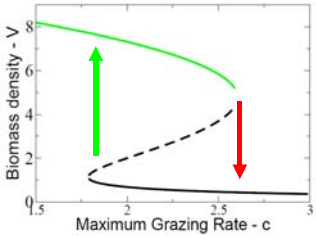
Model for vegetation collapse

- Dynamics of vegetation in semi-arid regions under (stochastic) grazing (May, 1977)

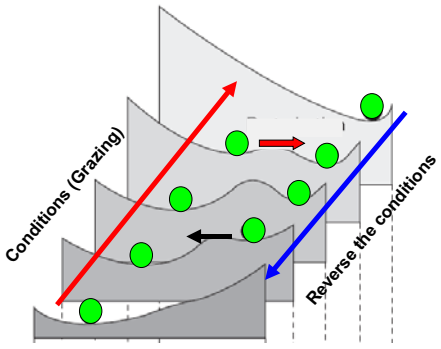
V: Biomass density

$$\frac{dV}{dt} = \underbrace{rV\left(1-\frac{V}{V_c}\right)}_{\text{Logistic Growth}} - \underbrace{(c+\sigma_V\eta_V(t))\frac{V^2}{V^2+V_0^2}}_{\text{Losses due to grazing}}$$

- Bifurcation diagram for the deterministic model:



Intuitive landscape picture



For simple models. we can define the landscape potential U(x) as follows

Ref: Horsthemke and Lefever, 1984

$$\dot{x} = f(x) + g(x)\eta(t)$$

$$U(x) = -\int_{x_0}^{x} \frac{f(x) - \sigma^2 g(x)g'(x)}{g(x)^2}\,dx.$$

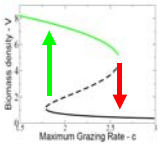
Scheffer et al, Nature, 2001: 591-596; Holling 1973; May 1977

Indicators of proximity to threshold

Far from threshold

Close to a threshold

Flattening of landscape



- Increase in the recovery rate and variance
  - C. Wissel (1984); Held & Kleinen (2004); van Nes & Scheffer (2007)
  - Variance: S K Ma (1976); Kleinen et all, 2003 ; Oborny, et al (2005); Carpenter & Brock (2006)
- "Critical fluctuations" of a phase transition
  - S K Ma (1976) Modern theory of critical phenomena

$$\dot{x} = -\alpha x + \eta(t) \qquad \alpha \to 0$$

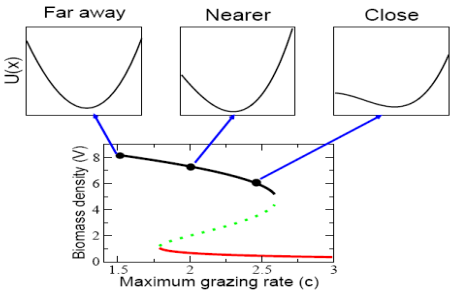
as we go towards bifurcation.

$$C(\tau) = \langle x(t)x(t+\tau)\rangle = \frac{\sigma^2}{2\alpha}e^{-\alpha|\tau|}$$

$$\tau_c = 1/\alpha$$

Devising a new indicator

- Have a closer look at the potential landscape picture:



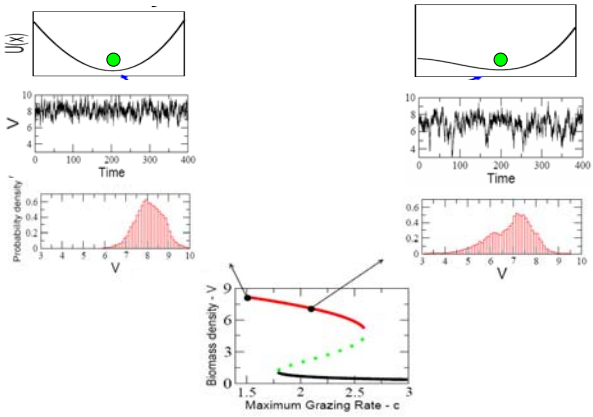
$$\dot{x} = -\alpha x + \beta x^2 + \eta(t)$$

$$U(x) = \frac{\alpha}{2}x^2 - \frac{\beta}{3}x^3.$$

- Pronounced **asymmetry** around the stable state
  - Nonlinear effects

V Guttal and C Jayaprakash, 2008. Ecology letters, Vol. 11, 550-560

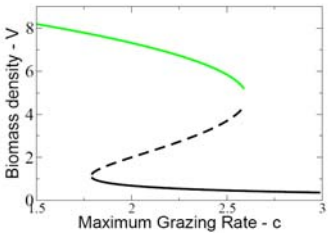
Asymmetry in time series distribution

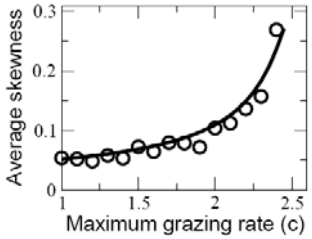


V Guttal and C Jayaprakash, 2008. Ecology letters, Vol. 11, 550-560

Skewness: Quantifying asymmetry

$$\gamma = \frac{\int (x-\mu)^3 P(x)dx}{\sigma^3}$$





Skewness increases as the threshold is approached!

An increasing skewness, or more generally a changing skewness, can be an earnly warning signal of approaching a regime shift.

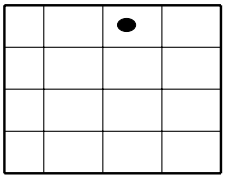
V Guttal and C Jayaprakash, ***2008.*** Ecology letters, Vol. 11, 550-560

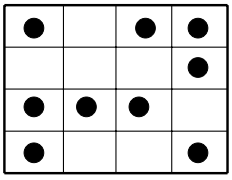
Outline

- Given a *time series data* of a state variable at one site:
  - How do we assess proximity to a transition?
  - Quick review of other indicators
  - ***Changing skewness*** is proposed to be an indicator

- **Given a time series of full *spatial data:***
  - Patches, ***Spatial variance, Spatial skewness*** can provide early warning signals.
  - Qualitative and quantitative improvements with spatial data

Ecosystem in space

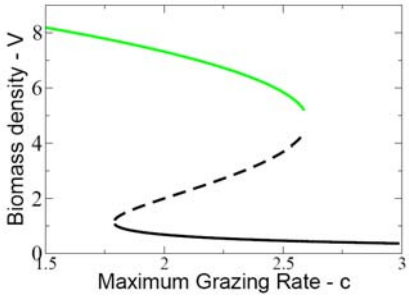




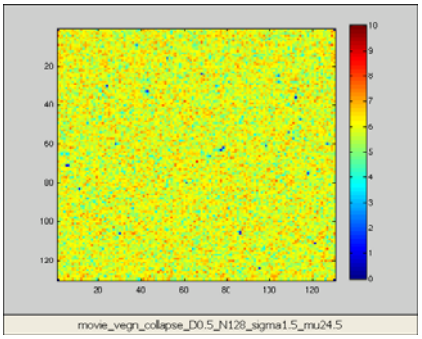
Model for collapse of vegetation: with **space**

$$\frac{\partial V(x,t)}{\partial t} = rV(1-\frac{V}{V_c}) - (c+\eta_c(x,t))\frac{V^2}{V^2+V_0^2} + D\nabla^2 V(x,t)$$

Diffusive seed dispersal

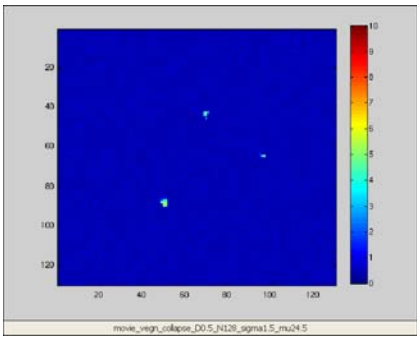


Regime shift with space



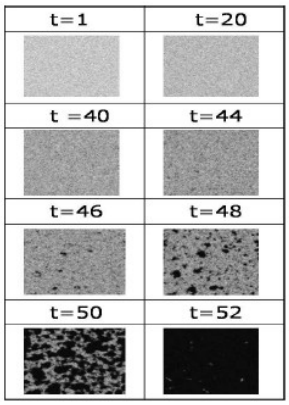
Slowly increase the grazing rate towards the threshold.
Red-Yellow: High vegetation density
Blue :Low vegetation density

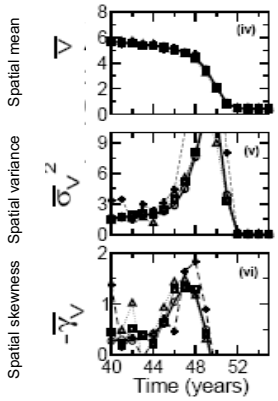
Animation of regime shift.



- Patch dynamics by itself can be an indicator of regime shift.
  - Studied in population dynamics literature
- Can we quantify?

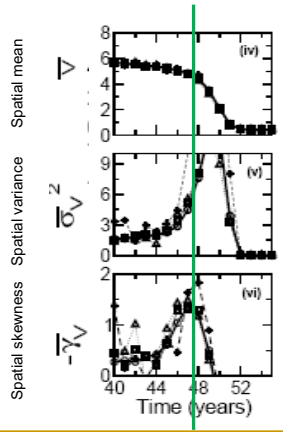
Spatial indicators:





Spatial indicators

- By year 48
  - Mean begins sharp decreasing.
- By year 46
  - 300% increase in ***spatial variance***
  - 0 to 1 change in ***spatial skewness***
- Temporal correlations between variance and skewness
  - ***Peaking*** spatial skewness with continued increase in spatial variance (by year 48)
  - Serves as an additional indicator

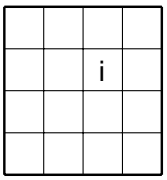


Different dispersal kernels

- **Include generic kernel of dispersal**: k(x,y)

$$\frac{\partial V(x,t)}{\partial t} = rV\left(1-\frac{V}{V_c}\right) - (c+\eta(x,t))\frac{V^2}{V^2+V_0^2} + \int_\Omega k(x,y)[V(y,t)-V(x,t)]dy$$

- **Mean field approximation**:
  - Neighborhood -> Effective medium
  - Obtain the effective medium through self-consistent equations

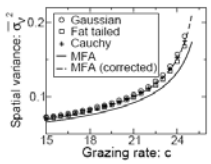


$$E(v) = \int v\, P_{mft}(v, E(v))dv$$

$$P_{mft}(v) = \frac{1}{\mathcal{N}}\exp\left[\frac{2}{\sigma_d^2}\int_{v_0}^{v} du \frac{f(u)-\sigma_d^2 g(u)g'(u)-(1-k_0)(E(v)-u)}{g(u)^2}\right]$$

- **Results are independent of the dispersal kernel and the spatial dimensions within MFA** (under some simple conditions)

Comparison of MFA with numerical simulations



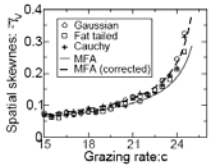


Table 3.1: Redistribution kernels and their properties

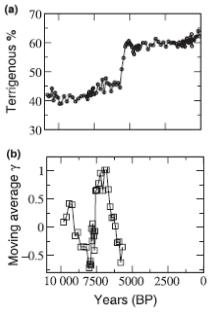
Name	k(x)	Moments	$\hat{k}(s)$ (Generating function)
Gaussian	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$	$k_2=\sigma^2,\ k_4=3\sigma^4$	$e^{\frac{\sigma^2 s^2}{2}}$
Exponential	$\frac{1}{2\theta}e^{-\lvert x\rvert/\theta}$	$k_2=2\theta^2,\ k_4=24\theta^4$	$(1-\theta^2 s^2)^{-1}$
Fat tailed	$\frac{1}{4\alpha}e^{-\sqrt{\lvert x\rvert/\alpha}}$	$k_2=5!\alpha^2,\ k_4=9!\alpha^4$	Does not exist
Heavily fat-tailed	$\frac{1}{\pi}\frac{\beta}{\beta^2+x^2}$	Does not exist	Does not exist

Summary of early warning signals:

- Devised new early warning signals
  - Using simple models of bistable ecological models.
  - Suggested quantities are easy to measure.
  - Have the potential for applications in many systems.
- Given time series data
  - Increase in variance, ***Changing skewness*** and Increase in recovery time
- Given spatial data at regular time intervals (e.g. year).
  - *Spatial variance* and ***spatial skewness***
  - ***Peaking skewness with increasing spatial variance***
- Further work
  - False alarms, apply to data, etc.
- Acknowledgements
  - NSF Grant DEB-0410336 and Presidential Fellowship, OSU

Sahara vegetation collapse

- **Vegetation collapse in Sahara**
  - ~5500 yrs back



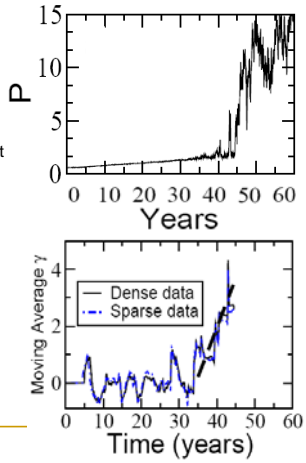
- **Skewness Behavior**
  - Statistically insignificant ☹
  - Similar issues with variance

P. deMenocal et al. / Quaternary Science Reviews 19 (2000) 347–361

Guttal and Jayaprakash, ***Ecology letters, 2008***

Simulation results

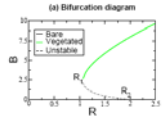
- Regime shift at year 45
- Calculate ``Moving Skewness''
  - Skewness for data from previous five years
  - Shows small fluctuations far from a regime shift
  - A ***sustained increasing trend*** (in comparison to background trend) occurs prior a regime shift.
- Nearly 100% increase in skewness by year 40 - five years in advance
- Data constraint
  - 100 (dense)
  - 33 measurements per year (sparse)

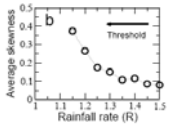


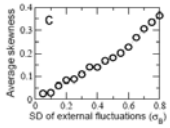
A nonpotential system

- **Effective potential need not exist multivariable systems**
  - Include soil water dynamics (w) *(V Guttal ad C. Jayaprakash, 2007)*

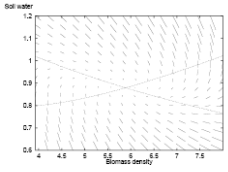
$$\frac{dw}{dt} = \underbrace{R}_{\text{Rainfall}} - \underbrace{\alpha w}_{\text{Evaporation}} - \underbrace{\lambda w B}_{\text{Plant uptake}} + \sigma_w \eta_w(t) \qquad \frac{dB}{dt} = \underbrace{\rho w B(1-\frac{B}{wB_c})}_{\text{Logistic Growth}} - \underbrace{\mu \frac{B}{B+B_0}}_{\text{Loss due to grazing}} + \sigma_B \eta_B(t)$$

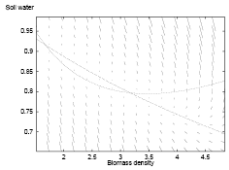






- **Indicator was based on potentials**
  - Most results hold for more complex model systems.
- **Flow diagram**





Horsthemke and Lefever, 1984; Gardiner, 2003; V Guttal and C Jayaprakash, 2007, Ecological Modeling