A spatial population model with random obstacles

4th Cornell Probability Summer School

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Mild obstacles

Model.
Let ω be a Poisson point process (PPP) on $\mathbb{R}^d$ with intensity $\nu > 0 \sim \mathbf{P}$.
$a > 0$ and $\beta_2 > \beta_1 > 0$ fixed.

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Given ω, we define P^ω as the law of the (strictly dyadic) BBM on $\mathbb{R}^d$, $d \geq 1$ with spatially dependent branching rate

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Total mass process: $|Z|$.

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Questions can be asked in 2 diff. ways: annealed and quenched sense.

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Population moves in space and reproduces by binary splitting, but randomly located reproduction-suppressing areas modify the growth.

(ii) **Fecundity selection** (Genetics):
Reproduction and mutation. Certain randomly distributed genetic types have low fitness: even though they can be obtained by mutation, they themselves do not reproduce easily, unless mutation transforms them to different genetic types.
['Space' = space of genetic types rather than physical space.]

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- **Population dynamics setting:** Is there a preferred spatial location for the process to populate?
- **Genetic setting:** Existence of a certain kind of genetic type that is preferred in the long run that lowers the risk of low of fecundity caused by mutating into less fit genetics types?

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Related phenomenon in marine systems: hypoxic patches form in estuaries because of stratification of the water.
⇛The patches affect different organisms in different ways but are detrimental to some of them. They appear and disappear in an effectively stochastic way.

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- FORESTS where trees sometimes fall creating gaps (which have various effects on different species but may harm some)
- AREAS OF GRASS or brush which are subject to occasional fires — burned areas can be expected to less suitable habitats for at least some organisms.

Result on population size

Define *average growth rate*:

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Theorem

On a set of full $\mathbf{P}$*-measure,*

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That is, loosely speaking,

$$r_t \approx \beta_2 - c(d,\nu)(\log t)^{-2/d}.$$

Result on spatial spread

An upper estimate on the spatial spread. The order of the correction term is *larger* than the $\mathcal{O}(\log t)$ term in a result of Bramson, namely it is

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Theorem

β_1 plays no role and, loosely speaking, at time t the spread of the process

$$\leq t\sqrt{2\beta_2} - c(d,\nu)\sqrt{\frac{\beta_2}{2}} \cdot \frac{t}{(\log t)^{2/d}}.$$

Problem (Shape of branching tree)

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Problem (More general branching)

What happens when dyadic branching is replaced by a *general* one? E.g. critical branching. Let $\beta_1 = 0$. It is still true (nontrivial) that

$$P^{\omega}(\text{extinction}) = 1, \ \ a.e. \ \omega \in \Omega.$$

Q: What is the order of the tail $P(\tau_{\text{ext}} > t)$?

THANK YOU FOR YOUR ATTENTION!