1. Homework (due Friday, September 08)
   Page 17: 1; Page 27: 7, 8, 10, 11; Page 29: 14, 15; Page 30: 19; Page 32: 2, 4 (facultative, try it only if you have time).

2. Homework (due Friday, September 15)
   Pages 41-42: 8, 9; Page 45: 17; Page 49: 24; Page 50: 27.

3. Homework (due Friday, September 22)
   Page 54: 4, 5; Page 58: 6, 7; Page 71: 18, 19.

4. Homework (due Friday, September 29)
   No homework due Friday, October 06.

5. Homework (due Friday, October 13)
   Page 88: 4; Page 95: 16; Page 118: 9, 10; Page 150: 6; Page 151: 9(d); Page 154: 10.

All of the above homeworks have been chosen from the ODE book of Brauer and Nohel. From now on, we will only use the PDE book of Zachmanoglou and Thoe !!! There is no homework due Friday, October 20.

6. Homework (due Friday, October 27)
   Page 173: 1.2; Page 179: 2.7 (by separation of variables in rectangular coordinates, one just means looking for solutions of type $u(x, y) = v(x)w(y)$). And the following two additional problems:

**Problem 6.1.** Show that in three dimensions, as long as $x \in S(x_0, r)$, one has

$$\frac{\partial}{\partial n} \left( \frac{1}{|x - x_0|} \right) = -\frac{1}{r^2},$$

where $n$ is the exterior normal to the sphere (we used this fact in class, when we proved the representation theorem in $\mathbb{R}^3$).

**Problem 6.2.** Show that if $u(x, y)$ is a harmonic function in the plane, then $v(r, \theta) := u(r \cos \theta, r \sin \theta)$ satisfies the equation (called the Laplace equation in polar coordinates):

$$\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial v}{\partial r}) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0$$
7. Homework (due Friday, November 03)

Page 196: 5.6; Page 205: 7.5. And the following two additional problems:

**Problem 7.1.** Show that if $z$ is the complex number $z = re^{it}$, then one has

$$Re\left(\frac{1 + z}{1 - z}\right) = \frac{1 - r^2}{1 + r^2 - 2r \cos t}.$$  
(We used this, when we derived the formula for the Poisson kernel)

**Problem 7.2.** Let $\Omega \subseteq \mathbb{R}^3$ be a “nice domain” and $u : \Omega \to \mathbb{R}$ be a $C^2(\Omega)$ function. Assume also that $u$ has the Mean Value Property which says that for every $x \in \Omega$ and $r > 0$ such that $B(x,r) \subseteq \Omega$, one has

$$u(x) = \frac{1}{|S(x,r)|} \int_{S(x,r)} u(\alpha) d\alpha.$$  

(a) Show that, by using the simple change of variable $\alpha = x + r\sigma$, one can also write $u(x)$ as

$$u(x) = \frac{1}{|S(0,1)|} \int_{S(0,1)} u(x + r\sigma) d\sigma$$  
for every $r > 0$ as before.

(b) Use the symmetry of the unit sphere to show that

$$\int_{S(0,1)} \sigma_1 \sigma_2 d\sigma = \int_{S(0,1)} \sigma_1 \sigma_3 d\sigma = \int_{S(0,1)} \sigma_2 \sigma_3 d\sigma = 0$$

and also that

$$\int_{S(0,1)} \sigma_1^2 d\sigma = \int_{S(0,1)} \sigma_2^2 d\sigma = \int_{S(0,1)} \sigma_3^2 d\sigma = \frac{4\pi}{3}.$$  

(c) For a fixed $x \in \Omega$ denote by $f_x(r)$ the function given by

$$f_x(r) = \frac{1}{|S(0,1)|} \int_{S(0,1)} u(x + r\sigma) d\sigma$$

for $r > 0$ as before. Calculate carefully $\frac{d^2}{dr^2}(f_x(r))$ (using also the information from (b)) and show that

$$\lim_{r \to 0} \frac{d^2}{dr^2}(f_x(r)) = \frac{1}{3} \nabla^2 u(x).$$

(d) Deduce from here that $u$ must be a harmonic function.

8. Homework (due Friday, November 10)

Page 221: 8.5, 8.10(b, c, d); Page 226: 9.1, 9.3.

9. Homework (due Friday, November 16)

Page 316: 8.5, 8.7, 8.9.
10. Homework (due Friday, November 30)
Page 273: 2.1, 2.3, 2.4; Page 279: 3.1(a); Page 283: 4.1, 4.2; Page 290: 5.2.

11. Homework (just to practice for the final exam)
Page 342: 2.1, 2.4, 2.5, 2.6; Page 347: 3.5