Exercise 4.1. Prove that a CAT(1) piecewise spherical simplicial complex is metrically flag.

Exercise 4.2. Show that every properly proper action is proper, and that every proper action on a proper metric space is properly proper.

Exercise 4.3. Let $G$ act properly and cocompactly on the length space $X$. Show that $X$ is complete and locally compact.

Let $G = \langle \Sigma \rangle$ be a finitely generated group. For every element $g \in G$, the translation length (with respect to $\Sigma$) is defined as

$$\tau(g) := \lim_{k \to \infty} \frac{|g^k|}{k}.$$

Exercise 4.4. Show that the translation length of a group element is well defined, i.e. the limit exists and is independent of the generating set $\Sigma$.

Exercise 4.5. Let $G$ be virtually $\mathbb{Z}^n$, and let $H$ be a subgroup of $G$ that is isomorphic to $\mathbb{Z}^n$. Show that $H$ has finite index in $G$. (Hint: Consider a finite index $\mathbb{Z}^n$ inside $G$ and the action of $H$ on $G/\mathbb{Z}^n$.)

Exercise 4.6. Let $G$ be finitely generated. Show that $G$ is virtually abelian provided the commutator subgroup $[G, G]$ is finite. (Hint: Let $H$ be the centralizer of $[G, G]$ in $G$ and show that the center of $H$ has finite index in $H$ and that $H$ has finite index in $G$.)

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.