Math 758 – *Your* Favorite Groups (homework 2, due Feb 12)

Exercise 2.1. Check that the the geometric representation does exist, i.e., check that the automorphisms ρ_s satisfy the defining relations of W.

Exercise 2.2. Show that W is finite if the bilinear form $\langle -, - \rangle_M$ is positive definite.

Exercise 2.3. Show that if W is finite, then there is a unique bilinear form $\langle -, - \rangle$ on V characterized by the following properites

1. $\langle -, - \rangle$ is positive definite.
2. All besis vectors e_s have unit length.
3. The action of W preserves $\langle -, - \rangle$.

Moreover, this bilinear form is $\langle -, - \rangle_M$.

Exercise 2.4. A Coxeter system is called irreducible, if there is no generator that commutes simultaneously with all the others. Classify all irreducible Coxeter systems over three generators whose Coxeter groups are finite. (Hint: You should recover descriptions of the Platonic solids along the way; in fact, the existence of the Platonic solids can be derived from this classification.)

Exercise 2.5. Show that for every $w \in W$,

$$wC \subseteq U_s^+ \text{ if and only if } |sw| = |w| + 1$$

and

$$wC \subseteq U_s^- \text{ if and only if } |sw| = |w| - 1.$$

Exercise 2.6. Infer from (2.5) that the geometric representation is faithful.

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.