Math 739 – Important Groups
Homework 3, due Feb 15

Exercise 3.1. The usual category theoretic nonsense proves uniqueness of direct limits for free. Show that direct limits exist in the category of groups and homomorphisms.

Exercise 3.2 (Schreier's Index Formula). Let G be a subgroup of F_n of finite index s. Prove that G is isomorphic to $F_{s(n-1)-1}$.

Exercise 3.3. Suppose φ and ψ are two hyperbolic automorphisms of a tree T whose axes have a finite intersection. Show that sufficiently high powers φ^k and ψ^l generate a free group.

Exercise 3.4. Show that a virtually solvable group cannot contain a non-abelian free group.

Exercise 3.5. A <u>paradoxical decomposition</u> of a group G is a partition

$$G = S_1 \uplus \cdots \uplus S_r \uplus T_1 \uplus \cdots \uplus T_s$$

such that there are group elements $g_1, \ldots, g_r$ and $h_1, \ldots, h_s$ such that

$$G = g_1 S_1 \cup \cdots \cup g_r S_r$$

and

$$G = h_1 T_1 \cup \cdots \cup h_s T_s$$

Prove that F_2 has a paradoxical decomposition.

Exercise 3.6. Find an *efficient* algorithm to solve the conjugacy problem.

It suffices to solve, on the average, half of the problems correctly.