Math 661 – Geometric Topology (homework 10, due Nov 18)

Exercise 10.1. Show that for any three angles α_1, α_2, and α_3 with $0 < \alpha_i < \pi$ and $\alpha_1 + \alpha_2 + \alpha_3 < \pi$ there exists a triangle in the hyperbolic plane with these interior angles. Moreover, this triangle is unique up to isometry.

Exercise 10.2. Let Σ be a surface, let P be a polygon, and let $f : P \to \Sigma$ be a map that realizes Σ by identifying the edges of P in pairs. Prove that the universal cover $\tilde{\Sigma}$ is naturally tiled with copies of P that intersect only along their boundaries.

Exercise 10.3. Let P be an n-polygon that has positive interior angles assigned to its corners. Prove that this polygon can be drawn in the hyperbolic plane provided the angles add up to strictly less then $(n-2)\,\pi$.

Exercise 10.4. Let the Poincaré disc

$$\mathbb{D}^2 := \left\{(x,y) \,\middle|\, x^2 + y^2 < 1\right\}$$

be the unit disc endowed with the Riemannian metric

$$\mathrm{d}\,s^2 := 4\frac{\mathrm{d}\,x^2 + \mathrm{d}\,y^2}{\left(1 - x^2 + y^2\right)^2}.$$

Show that $\mathbb{D}^2$ and $\mathbb{H}^2$ are isometric.

Exercise 10.5. Show that geodesics in the Poincaré disc look like circles perpendicular to the unit disc.

Exercise 10.6. Let P and Q be two points in the hyperbolic plane. Give ruler and compass constructions for the geodesic through P and Q in the upper half plane model and in the Poincare disc model.

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.