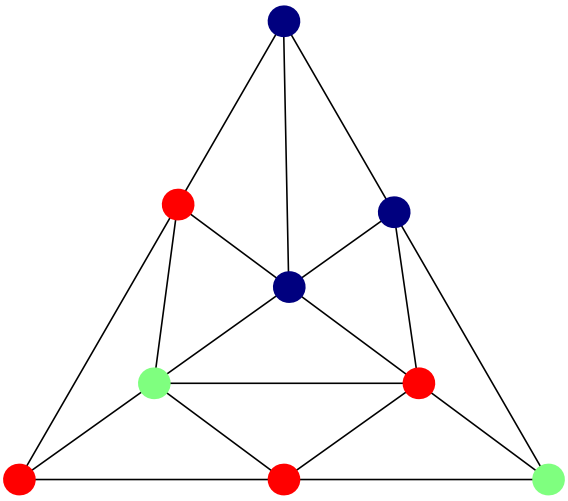
Math 661 – Geometric Topology (homework 3, due Sep 27)

Exercise 3.1. Assume, you have a triangle which is triangulated, i.e., it is tiled by triangles that intersect only in vertices and edges such that a vertex of one triangle does not lie in the interior of an edge of another triangle. Assume that the vertices are colored in red, blue, and green such that the following conditions are satisfied:

1. The three vertices of the big triangle are given pairwise different colors.
2. Along each of the three edges of the big triangle, only two colors are used for the vertices in this edge.

Prove that you will always find a tricolored triangle in the subdivision.



Exercise 3.2. Use (3.1) to prove Brouwer's fix point theorem for the 2-dimensional disc: Every continous map $f : \mathbb{B}^2 \to \mathbb{B}^2$ has a fixed point.

Exercise 3.3. Find a triangulation of the torus that uses as few triangles as possible.

Exercise 3.4. Show that every (G, X)-atlas for M is contained in a unique maximal (G, X)-atlas.

Exercise 3.5. Show that the full isometry group of Euclidean m-space is rigid for $\mathbb{E}^m$.

Exercise 3.6. Find two non-similar Euclidean structures on the 2-dimensional torus.

Each problem is worth 5 points, but you can earn at most 20 point with this assignment.

Late homework will not be accepted.