# Request For Comments on a Family of Classic Guitars

Robert E. Terrell *

May 16, 2005

draft

The current version of this document can be found at

http://www.math.cornell.edu/~bterrell

## 1 Introduction

The classic guitar is often played in quartets of nearly identical instruments. It is possible that there have been discussions or attempts to introduce guitars of other sizes to increase the repertoire and tonal possibilities of the instrument. I do not know, but these are my thoughts about it, and I would welcome ideas from other people.

This document is a description of a family of four instruments which includes the classic guitar as the third largest. The purposes of this study are:

- To help me understand the construction and resonances of the guitar better by focusing on the sizes of its parts relative to the tones and pitches desired.
- To solicit comments from musicians on the features described here. That is to say, if people other than myself are interested in this subject, I would like to hear from them.

*Send comments to: bterrell@math.cornell.edu

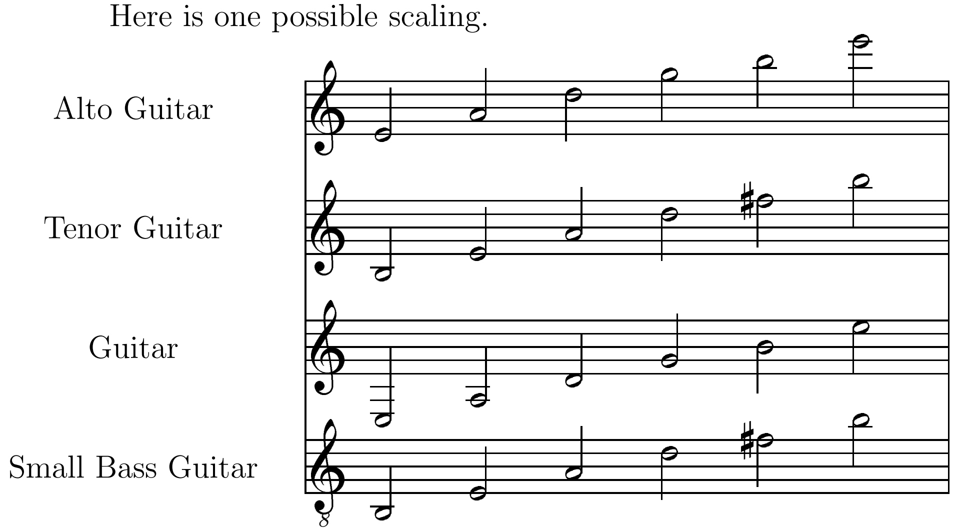
I will build a set of these instruments to learn more about them. I do not know whether others have done or have discussed doing the same thing. I do not know whether anybody actually wants guitars of various sizes. I do think it will be fun to play them.

Information on the resizing of musical instruments is available from the literature on the violin octet [2] as well as from general studies of vibration [1], solid mechanics [4], and musical acoustics [3].

This document records a lot of data from these various sources, for easy access, without trying to organize it all perfectly.

## 2 Tuning

Here is one possible scaling.




The guitar notes are shown an octave above their actual location, as usual, so the “8” subscript on the small bass guitar staff indicates still one more octave decrease. Although bassists are accustomed to reading from the bass clef, guitarists might not be. I have elected here to use these conventions on the assumption that the players of all four instruments are guitarists. One might go further and transpose as is done in the saxophone family, but that seems inadvisable particularly in the beginning of this study when the very pitches of the instruments are still speculative.

All the instruments have six strings at the same intervals as the guitar. The alto (mezzo?) is shown tuned an octave above the guitar, the tenor (alto?) a fifth above the guitar, and the small bass a fourth below the guitar.

It is possible that one can tune down a fifth to AEDGCA, but I think this is pitched too low to produce adequate volume from an instrument having roughly the same size as a guitar. The tuning has to go down to C at least so that cello parts may be played. If it were tuned AEDGCA, then that would be as low as the small bass of the violin octet and the Mexican guittarron, in some tunings. Unlike the guittarron this small bass guitar is to be played seated, in three-point contact like the guitar, so the back can vibrate freely. This improves the bass response and so maybe there is hope for that lower tuning, but I'm assuming for now that we can only go down a fourth from the guitar.

## 3 Music

On this I am not qualified to say anything. Perhaps there are string quartets that could be performed on this family.

## 4 Practical Matters

The guitar maker must purchase strings, cases, and tuning machines. All else he makes. The standard tuning machines can be used on all these instruments, but they might look funny on the alto. We won't worry about cases yet. One can make cases if one has to.

The lowest string on the small bass will likely be a nylon core metal wrapped (or vice-versa) string designed for the so-called "acoustic bass guitar" or perhaps a string designed for a 3/4 sized string bass. The highest two strings on the alto might be kevlar fishing line or similar. Historically one knows that when gut was replaced by nylon, the first material used was fishing line. All other strings can be standard classic guitar strings of various weights, but not always in the positions they were designed for.

## 5 Playing Position

The playing position is seated, as for the guitar. Alto and tenor might require a support to the leg similar to what is used by many players of the guitar. Possibly the small bass will go to the right leg rather than the left, depending on the length of strings and body of this instrument.

# 6 Violin Octet Scaling

Frequency ratios are about 3:2 (really $2^{\frac{7}{12}}$) for a fifth, such as $C_2$ (65.5 Hz) to $G_2$ (98 Hz), and about 4:3 ($2^{\frac{5}{12}}$) for a fourth, say $G_2$ (98 Hz) to $C_3$ (131 Hz).

The lowest notes of the violin octet family are $E_1$, $A_1$, $C_2$, $G_2$, $C_3$, $G_3$, $C_1$, and $G_1$. For the violin octet the body length ratio is about 1:1.2 between the baritone violin and small bass violin whose lowest notes are a third apart, and from the mezzo violin to the treble, an octave apart, the body length ratio is about 1.4. The widths are scaled by approximately the same ratios. The rib heights scale by 1.75 regardless of the interval, except for 1.28 from the small bass violin to the contrabass violin. The string lengths are not scaled by such simple ratios. All string length ratios between adjacent violins are between 1.08:1 and 1.2:1. Plate thicknesses and air volume in the body are critical to get the resonances right. Plate thickness possibly does not scale as much as the plantilla, if we can judge from the violin basses. But why does the treble violin have such thick plates, and what does that imply for our alto? It is that the treble violin is already so small to hold and play, that the body is really too long acoustically? If so one compensates by increasing the plate thickness.

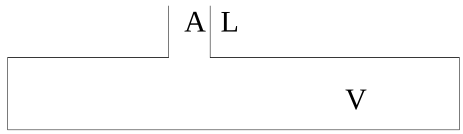
# 7 Acoustic Theory

There are mathematical models for vibrations of strings, plates, and air masses, which can shed some light on the rescaling of musical instruments. Like all mathematical models, the solvable cases are simpler than the reality, but useful anyway. For each of these items, some estimates can be made of the effects of resizing.

## 7.1 Helmholtz resonantor

This is the first approximate model for the tap tone you hear if you tap the soundboard of the guitar between the bridge and the tail. This tone is sometimes described as that heard by blowing obliquely into the soundhole, but tapping seems to give the same thing more reliably on my guitars. However, I don't know the extent to which this tap tone can really be said to be Helmholtz, or an interaction of that with plate modes.

The Helmholtz frequency for vibrations of a mass of air in the neck of a rigid cavity against the compression of air inside is given by




$$f_H = \frac{c}{2\pi}\sqrt{\frac{A}{VL}}$$

Here $L$ is the length of the bottle neck, and $c$ the speed of sound.

`http://www.phys.unsw.edu.au/~jw/Helmholtz.html`

In guitars and violins, the role of neck area is played by the area of the soundholes, while the role of $L$ is subject to discussion. A value for $L$ can be estimated in guitars by damping the strings and holding one hand at various distances above the soundhole while tapping the soundboard with the other hand. The distance at which little airflow is felt then suggests the outer limits for $L$. This is decribed in the web page above.

Now look at the effect of rescaling the Helmholtz resonator. The scaling in the violin octet uses one factor, say $s = 1.25$ for the plantilla length and width, and another factor $r = 1.75$ for the rib height, when lowering by a fourth. Suppose we use the same plantilla scaling for the guitar family. But the rib factor is out of the question for the small bass, because you will never get your arm around a guitar 175mm thick. Let us use $r = 1.20$, because we know there are many players of "jumbo" guitars who can handle a thickness of 125mm or more, though it looks awkward for small guitarists. We can also taper the body a little to help with this problem if needed. (In fact, the three point contact playing position allows a thicker instrument than does the standing, back muffled, position, because the right forearm reaches more easily to the inclined top.)

So we try $r = 1.20$.

To reduce by a fourth, from the guitar tap tone at $G_2 = 98$ Hz to a possible small bass tap tone at $D_2 = 74$ Hz we need 3/4 the frequency. Is this achieved? The volume is multiplied by $s^2 r$, and the neck area by $s^2$, assuming that the soundhole is scaled with the soundboard. Then the Helmholtz frequency is multiplied by

$$\sqrt{\frac{s^2}{s^2 r}} = \frac{1}{\sqrt{1.20}} = 0.913.$$

That is not a sufficient reduction. But note the soundhole area effect: if the soundhole diameter is not changed at all from that of the guitar, then the Helmholtz frequency is multiplied by

$$\sqrt{\frac{1}{s^2 r}} = \frac{1}{\sqrt{(1.25)(1.25)(1.20)}} = 0.73.$$

So tentatively, if the small bass guitar has ribs 1.2 times those of the guitar, plantilla scaled by 1.25, and no change to the soundhole, then we can lower the Helmholtz frequency by a fourth. What is lost by this? Sound volume might be lost because the flow of air has been obstructed to reduce the frequency, consequently energy is wasted into turbulence. We can try it anyway and see what happens.

For the tenor: tap tone $D_3$ 148 Hz. Write $s_{\text{len}} = .75$ for the body length scaling, and $s_{\text{wid}} = .78$ for the width scaling. The width is 10mm more than would result from scaling the guitar plantilla by .75, to allow for the relatively wide fingerboard. Keeping the soundhole size 84mm of the guitar, so we can still reach in, we need

$$\frac{1}{s_{\text{len}} s_{\text{wid}} r} = \left(\frac{3}{2}\right)^2$$

giving $r = .76$.

For the alto: tap tone $G_3$ 196 Hz. Use $s_{\text{len}} = .65$ and $s_{\text{wid}} = .66$. The width is 20mm more than would result from scaling the guitar plantilla by .66. The length is 24mm longer, the distance from 14th to 16th fret, with all this added to the upper bout. Otherwise the soundhole is too close to the neck-body joint. Keeping the soundhole size again, we need

$$\frac{1}{s_{\text{len}} s_{\text{wid}} r} = (2)^2$$

giving $r = .58$.

## 7.2 Strings

The vibrations of idealized strings are at frequency

$$f_{\text{string}} = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

and all integer multiples, where $T$ is the tension, $L$ the length, and $\rho$ the mass per length. Now let us find a sixth string for the small bass guitar. We start with a 34 inch "acoustic bass" D string at 74 Hz. Reduce the length to say 700mm (classic guitar strings are long enough for this for the other five cases), and reduce the tension by a factor of $\tau$. The frequency changes from 74 to

$$74\frac{34(25.4)}{700}\sqrt{\tau} = 74\frac{864}{700}\sqrt{\tau} = 74(1.234567)\sqrt{\tau} = 91\sqrt{\tau}$$

For this to be $A_1$ (55 Hz) we have to take $\tau = (55/91)^2 = .36$, about one third the tension. That could be alright, as far as the feel of the strings goes. It is probably not loud enough. To get $B_1$ (62 Hz) we take $\tau = (62/91)^2 = .46$, about half the tension. This seems more likely. Trying instead the "acoustic bass" A string, we find 0.83 the tension.

D'addario strings and Berkley Big Game nylon:

| | J47 (gold normal) | | J45 (silver) | | J48 (hard) | | B.B.G. | |
|---|---|---|---|---|---|---|---|---|
| | tension (kg) | dia (mm) | (kg) | (mm) | (kg) | (mm) | (test) | (mm) |
| | | | | | | | 12# | .36 |
| | | | | | | | 15# | .38 |
| | | | | | | | 20# | .46 |
| | | | | | | | 25# | .48 |
| | | | | | | | 30# | .56 |
| | | | | | | | 40# | .61 |
| | | | | | | | 50# | .71 |
| E | 6.94 | .71 | 6.94 | .711 | 7.17 | .74 | | |
| | | | | | | | 60# | .82 |
| B | 5.26 | .82 | 5.26 | .818 | 5.44 | .85 | | |
| | | | | | | | 80# | .89 |
| G | 5.49 | 1.02 | 5.49 | 1.024 | 5.62 | 1.04 | | |
| D | 6.99 | .74 | 7.08 | .74 | 7.40 | .76 | | |
| A | 6.40 | .84 | 6.80 | .89 | 6.73 | .91 | | |
| E | 6.05 | 1.09 | 6.35 | 1.09 | 6.33 | 1.12 | | |

There are also "acoustic bass" strings from D'addario, phosphor bronze on steel core, in diameters 41-86(?). The sound is too treble I think. Fender "taped bass" string diameters: 58-72-92-110 and GHS: 50-70-90-105-125. D'addario has 3/4 string bass helicores in heavy or light gauge. LaBella has supernil nylon 4/4 string bass low tension. Note that the string bass and cello seem to use higher tension than guitars.

## 7.3 Plates

For isotropic plates without axial loading it is

$$f_{\text{plate}} = \frac{\alpha}{L^2}\sqrt{\frac{D}{\rho_{\text{area}}}} = \frac{\alpha}{L^2}\sqrt{\frac{Eh^2}{12(1-\nu^2)\rho_{\text{vol}}}}$$

where $\alpha$ depends on conditions, $L$ is a length, $\rho$ is mass per area, and $D = \frac{Eh^3}{12(1-\nu^2)}$ etc. For a circular plate clamped at the boundary, $\alpha = 10.2$. A study of clamped circular plates immersed in a fluid is referenced in [4]. A prediction is made of the extent to which the frequency is lowered. Some information about plates with axial loading (string tension) occurs there also.

When rescaling plates: if you rescale a plate by factor $t$ in all three dimensions then the frequency goes as $t^{-1}$. If you keep the thickness fixed and rescale length and width then the frequency goes by $t^{-2}$. More specifically, with $h \to t_1 h$ and $L \to t_2 L$ we get $\rho \to t_1\rho$, $D \to t_1^3 D$, and $f \to t_1 t_2^{-2} f$.

## 7.4 Anisotropic Plates

Of course our plates are wood. Vibrations of undamped anisotropic thin plates are described by (mechanicians are taught to say "governed by", but musicians and woodworkers know better)

$$\frac{E'_x h^3}{12}\frac{\partial^4 w}{\partial x^4} + 2\left(\frac{E'' h^3}{12} + 2\frac{Gh^3}{12}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{E'_y h^3}{12}\frac{\partial^4 w}{\partial y^4} = -\rho h \frac{\partial^2 w}{\partial t^2}$$

with boundary conditions that the displacement $w = 0$ and $\nabla w = 0$ at the edges, for clamped plates. Perhaps guitar plates are clamped, probably not, but they certainly are not simply-supported. One doesn't know, except by feel, the extent to which the bindings and linings of the instrument affect the resonances. Sometimes violin ribs are viewed as mass loading rather than stiffener at the plate edges [2].

For the soundboard, the edges of the soundhole are free, which introduces new considerations I think.

Some elastic constants (psi) have been recorded in [4].

| | $E'_x$ | $E''$ | $G$ | $E'_y$ |
|---|---|---|---|---|
| birch plywood | 2 | .077 | .17 | .17 |
| maple plywood | 1.87 | .073 | .159 | .60 |

# 8 Pictures

The scaling shown in the figure has been influenced by holding some mockups of these scaled sizes, to begin judging the playing positions.

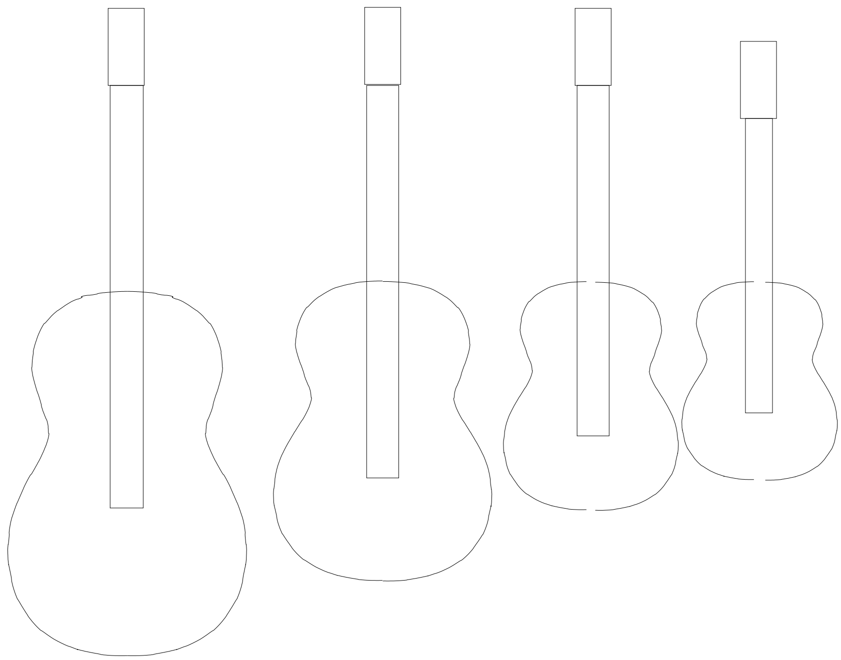
- body length– nominally $\frac{5}{4}$:1:$\frac{3}{4}$:$\frac{3}{5}$, modified however as explained earlier with the larger instruments relatively narrower

- string length– 729.6mm:650mm:579mm:487mm. This uses guitar EADGB strings and one larger for the small bass, with "G" tuned to F#. The strings are two frets longer and so have higher tension than in the guitar. The tenor guitar assumes that we use guitar ADGBE50lb strings shortened to the guitar's 2nd fret length, so all are under the same tension as in the guitar except for the "B" which we shorten to C# and then tighten to $D$. For the alto guitar there are several possibilities. I've drawn the case where we use the same strings as for the tenor, but shortened to the guitar's 5th fret position, and then tightened another whole tone. If that doesn't sound good we could use the 7th fret position, or go out to the 2nd fret position and DGBE50lb60lb strings. That is too long for the size of the body I think.

- approximately the same neck width, head size, and soundhole for all. 19 frets for all.

| | body wid | body len | body thick | plates | nut to body | join to boca |
|---|---|---|---|---|---|---|
| alto | .66 | .65 | 58mm | .86 | 270mm, 14th | 55mm |
| tenor | .78 | .75 | 76 | .88 | 321, 14 | 63 |
| guitar | 1 | 1 | 100 | 1 | 325, 12 | 105 |
| sm. bass | 1.1 | 1.25 | 120 | 1.03 | 320, 10 | 164 |

The rectangles shown enclosing the string areas run from the nut to the bridge saddle.



With these proportions, the instruments seem not as awkward to hold as I had expected. The length of the small bass requires that the right knee be moved further to the right than for the guitar. The tenor seems alright when the lower bout touches either the left or right leg, and the alto when the lower bout touches the left leg. Both tenor and alto are held with the neck more somewhat more elevated relative to the body than for the guitar.

## References


[1] Stephen Timoshenko. *Vibration Problems in Engineering.* Van Nostrand Company, 1937

[2] Carleen M. Hutchins, Editor. *Research Papers in Violin Acoustics, 1975–1993.* Acoustical Society of America, 1997

[3] Benade. *Musical Acoustics.* Dover

[4] S. Timoshenko Woinowsky–Kreiger. *Theory of Plates and Shells.* Engineering Societies Monographs, 1959