Teaching Statement

Tara E. Brendle

I have always loved teaching. Even though that statement has sometimes earned me strange looks in research-oriented departments, I have never seen teaching and research as opposing interests. I often return from teaching a class with a clear head and fresh energy to attack my own research. Teaching is another chance to be mathematically creative, a chance to see mathematics from a fresh perspective, as well as a chance to excite people about a subject which excites me.

I have been lucky enough to have had the opportunity as a young mathematician to teach a wide variety of courses, including:

- **Basic Mathematics** (in Columbia's School of General Studies)
- **Calculus** (for Arts and Sciences students, and also for engineers)
- **Linear Algebra**
- **Linear Algebra and Differential Equations**
- **Introduction to Topology** (point-set topology for math majors)
- **Knots and braids** (for first-year graduate students)
- **Seminar on mapping class groups** (for more advanced graduate students)

My students have run the whole range, from ambitious young pre-meds to film students, from math majors and graduate students already intent on a career in mathematics to 50-year-old women who were just going back to school to get a college degree and who hadn't taken a math class in more than 35 years.

With such diversity of students and subject matter, I have been surprised to learn that many of the same teaching strategies work well across the board. Though of course my approach differs in the details from class to class, it can be summed up as follows: I give a lot of examples, I emphasize definitions, I let the students create the mathematics whenever possible, and I try to make mathematics more accessible by being accessible myself, by sharing my own opinions of and experiences with the mathematics that I am teaching.

Naturally, there are important differences too. In contrast to teaching math majors and graduate students, one obvious challenge in teaching a course such as Basic Mathematics is that the students in the class tended to have had little previous success in mathematics, generally stated they "were just not math people", and were only taking the class because they had to in order to graduate. Yet it seems that few students, both those that classify themselves as "math people" and those that don't, have had mathematics presented to them as a subject in which one can display creativity. I like to turn the tables as often as possible in class and try to get the students to try to come up with new definitions, methods, or theorems before I state them myself.

For example, one of my favorite exercises with my Basic Math students (which I admit I borrowed from a colleague) was to ask them to imagine that I don't know what it means for a number to be "even", and they must explain to me what is meant by this term. Most often, they begin by saying something along the lines of, "An even number can be divided by 2." Then I ask, "So is the number 1 even? 1 can be divided by 2, with the result of $\frac{1}{2}$." They usually pause for a moment, then regroup, try again, and a discussion of the idea of being "evenly divisible" ensues. We go back and forth until we finally have an accurate definition (and often more than one) written down. This sort of exercise not only illustrates to the students that mathematics is useful as a way to learn sound, solid reasoning skills, but I find it also builds confidence in students who used to think they "just couldn't do math" – they see first hand that they are capable of creating mathematics themselves. It was incredibly rewarding to see so many students start to believe in their mathematical abilities even to the point where some kept taking math even after their requirement was fulfilled. I am extremely proud that my efforts with this group of students earned me Columbia University's Presidential Award for Graduate Student Teaching in 2002.

Of course, the same strategy works well for students at any level. I might play the same sort of game in order to elicit a reasonable notion of connectedness from my topology students, of isotopy vs. ambient isotopy from my graduate class, or of continuity from my calculus students. If the students figure something out for themselves rather than absorb it passively, the material sticks more often. The students are then permanently equipped with a new tool or skill and simultaneously develop a more sophisticated understanding of mathematics.

Another extra challenge when teaching non-math majors is convincing them that the experiences of *mathematicians* are relevant to their own; then they are more likely to relate to the mathematics itself. This can be as simple as admitting to calculus students that you find curve sketching problems to be as tedious as they do. Or it might mean that when a Basic Math student exclaims in class, "I hate this stupid stuff! How can you like doing this?" that I share with the class how I once mercilessly ridiculed a friend in college for wanting to be a math major. Then when I am telling them how I have always enjoyed simplifying algebraic fractions (because you get to cross out lots of "stuff"), or that I find working related rates problems to be fun (for no good reason), they actually have reason to believe me. More importantly, when I tell the Basic Math students that they do not have to like doing math, but that they are certainly capable of doing math, I think they start to believe me on that score too. The same goes for the calculus students who reluctantly start to accept the fact that the best way to get their "A" is to forget about Kaplanesque tricks and allow themselves to become engaged with the mathematics.

Sharing a bit of my own research can be fun too. My research of course fits in very easily in my topology class; I like to draw surfaces for them and explain how the topics they're currently learning fit in with active areas of research in mapping class groups. I am also looking forward to running a topology research project for REU students this coming summer at Cornell. I'll also share with my linear algebra students how I just had to compute the inverse of a matrix while working with representations of mapping class groups. Even though they do not understand all the words, they gain some insight into the vast importance of linear algebra in higher mathematics.

I have also found that an early emphasis on definitions is enormously beneficial to all students. Currently I give a weekly "definition quiz" to both my linear algebra students and my topology students. The linear algebra students come to realize the power of precision in language and learn to write proofs much earlier than when I previously did not give such quizzes. The topology students, who are more advanced, generally understand that they need to know definitions, but often overestimate their own ability to give precise definitions. The quizzes also help ensure they are keeping up with the syllabus.

Finally, I believe that the best way to learn mathematics is to have working examples in mind. As a teaching assistant working in the Math Help Room at Columbia long ago, I encountered a calculus student who could not give an example of a function which was continuous but not differentiable. I vowed then and there that no student of mine would ever walk into the Help Room with such a question. I try to provide as many illustrative examples of the theory as possible and to emphasize to students the importance and helpfulness of having examples at their fingertips.

While several of the courses I have taught had fairly standard, traditional syllabi, I was able to create my own syllabus for the two graduate courses I taught at Cornell. In the case of "Knots and braids", the lack of a good reference including all of the material I wanted to teach led to my collaboration with Joan Birman on "Braids: A Survey", which is now to be a chapter in the "Handbook of Knot Theory", ed. by W. Menasco and M. Thistlethwaite, and which Birman and I plan to expand into a book. The graduate students taking the course ended up being involved in this project, giving us constructive feedback on drafts and sometimes suggesting new proofs of various lemmas.

I was also able to shape the syllabus in the Berstein graduate seminar at Cornell, a course designed to involve graduate students in the presentation of course material. I designed a syllabus mostly consisting of research papers on mapping class groups of surfaces, my own area of expertise. After I gave one or two overview lectures, the graduate students volunteered to give talks on particular areas of interest to them. I would meet with them outside of class to discuss the papers they were reading, and then the students took turns presenting what they had learned to the class. It is a wonderful model; the students became much more involved, and (I believe) they learned a great deal more than in a traditional lecture-style class.

I have also taught in a summer program called EDGE (Enhancing Diversity in Graduate Education). The EDGE program, directed by Rhonda Hughes of Bryn Mawr College and Sylvia Bozeman of Spelman College, evolved from an REU-type summer program into a one-of-a-kind program for women in math, and in particular, for minority women, who are at the beginning stages of a graduate program in mathematics. The EDGE program is designed to identify and help women who are mathematically talented but who may be at risk for dropping out of graduate programs due to various social factors, such as having to navigate the transition from a small liberal arts college to a major research university, or feeling isolated as perhaps the only women and/or minority in their program. The EDGE program also helps all participants, both students and faculty, to build and maintain a network of mathematicians across the country.

I have been involved in the EDGE program since its beginning in 1998, first as a graduate student mentor and later as a short course instructor (in hyperbolic geometry) and as a visiting speaker. Having once struggled myself to make the transition to graduate work, it is particularly rewarding to help other young people successfully complete their programs.

Teaching is an important part of my life. It makes me a better mathematician, and I enjoy it immensely. The students do not always end up sharing my enthusiasm for algebraic fractions, but my hope is that they begin to view mathematics as a varied and dynamic subject which will have meaning for them no matter what course they choose in life, a subject in which their creative talents can flourish.