

3560 HOMEWORK EXERCISES, DUE 04/23/09

Work the following problems from Chapter 4:

- IV.3— 4, 7 (for #7, you should “check the claims” as indicated, but you only need to submit a proof of the final statement)
- IV.4— 2, 3 (omit part (e)), 4, 7

Hint for #3(f): review the definition and properties of the complex exponential function from section I.2.

Hint for #4: it may help to think about subgroups of \mathbb{Z} .

Additional problems:

- (1) Recall (Exercise II.3.8) that the *center* of a group G is the subgroup

$$C(G) = \{a \in G \mid ag = ga \text{ for all } g \in G\}.$$

- (a) Prove that $C(G)$ is a normal subgroup of G .
- (b) Prove that the center of D_8 is $\{\text{id}_{\mathbb{C}}, R_{\pi}\}$. Show that the quotient group $D_8/C(D_8)$ is isomorphic to D_4 .

(*Hint:* you may find it helpful to think about the homomorphisms $D_4 \rightarrow S_4$ and $D_8 \rightarrow S_4$ we saw in class, where the first one is given by labeling the corners of a square, and the second is given by labeling the diagonals of a regular octagon.)

- (2) Prove that the quotient group \mathbb{Q}/\mathbb{Z} is infinite, and that every element has finite order.