A: Find the highest power of 2 that divides 12!. Think about how you would find a general formula for giving the highest power of a prime number $p$ that divides $n!$.

B: Let $a + bi \neq 0$ be a complex number with $a, b \in \mathbb{Z}$. When does $a + bi$ have a multiplicative inverse $c + di$ (i.e. such that $(a + bi) \cdot (c + di) = 1$) such that $c, d \in \mathbb{Z}$ as well?

C: Find all rational solutions $(x, y)$ $(x, y \in \mathbb{Q})$ to the equation $x^2 + y^2 = 2$.

HINT: Assume $y - 1 = m(x - 1)$ and substitute for $y$ in the equation. Then solve for $x$ in terms of $m$ and then find a formula for $y$ in terms of $m$. Any $m \in \mathbb{Q}$ gives a rational solution. Conversely any pair of rational numbers $(x, y)$ with $x \neq 1$ satisfies an equation $y - 1 = m(x - 1)$ for a unique rational number $m$.

D*: If $\alpha = a + bi$, define the norm of $\alpha$, $N(\alpha) := a^2 + b^2$. Show that $N(\alpha) = (a + bi) \cdot (a - bi)$. $a - bi$ is called the conjugate of $a + bi$, denoted $a - bi$.

If $\alpha = a + bi$ and $\beta = c + di$, show that $\overline{\alpha \cdot \beta} = \overline{\alpha} \cdot \overline{\beta}$ and $N(\alpha \cdot \beta) = N(\alpha) \cdot N(\beta)$.