

**MATHEMATICS 401, SPRING 2006**  
**HONORS SEMINAR**

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**Course email address.** Mail sent to math401@math.cornell.edu will reach everyone in the class. Use this for questions of general interest, discussion, *etc.* This will not be available until the second week of classes.

**Course Description.** This year's seminar will consist of two components. The first will cover topics related to differential equations and transformation groups that leave them invariant. One of the main reasons for choosing this topic is because this year, towards the end of April, the mathematics department will organize a special series of lectures on this topic under the title **Lie theory days at Cornell**. There will be talks accessible to the undergraduates given by R. Howe from Yale, and A. Veselov from Loughborough. Since the students enrolled in 401 are strongly encouraged/expected to participate, we will focus on

*Huyghens' principle, PDE and dual reductive pairs, and Maxwell's equation.*

We will cover several chapters in the book by R. Howe and E. Tan, *Non-Abelian Harmonic Analysis*. The text is written at an elementary level, and will be on reserve in the library. The topics involve analysis, such as Fourier transforms and distribution theory, as well as algebra, relating to the representation theory of  $SL(2, \mathbb{R})$ . Y. Berest will give an introductory lecture. After that, the talks will be given by the students. We will also draw on material on Maxwell's equation from notes written by L. Gross.

The second component will cover a variety of topics chosen by the students. These require learning about a subject, making an oral presentation to the class, and writing a paper. There will be a strong emphasis on developing expository skills, with proper attention paid to history and motivation.

**Prerequisite.** The only prerequisite is *mathematical maturity*. I expect a lot of variation in the specific courses you have taken previously, and I expect lecturers to take this variation into account.

**Course requirements and grading.** Your grade will be based on both components, with emphasis on the big project. This might require reading the equivalent of a textbook, one or more research papers, and additional sources for history and background. You will then give one or two lectures and write a paper. I expect to meet with each of you frequently to help keep you on track.

**Working together.** For the first part I expect you to collaborate heavily. Your projects on the other hand are to be done individually. But I strongly encourage you to work together, in the following sense: Talk to each other about what you're doing, teach each other, ask each other probing questions, . . .

**Class participation.** I encourage you to interrupt the lecturer frequently. Say *I don't know what you're talking about; could you go a little slower?* Ask *Why would anyone want to do that?* Confess *I forgot what a line integral is; could you remind me?*

**Choice of topics.** I expect you to have committed to a topic by the end of the third week, but I am flexible. I emphasize, **you can talk about whatever you want, as long as it is mathematics and substantial.**

If you have trouble, consult me. Here is a list of topics that I might choose from if I were in your place. Some of them are related to the material of the first part. Bear in mind that they are suited to my tastes and they might be too hard (or too easy) for you. Many of them are interconnected.

- Fermat's last theorem
- quadratic forms
- character theory of finite groups and applications
- expander graphs
- the fast Fourier transform
- wavelets and applications
- numerical methods for ordinary differential equations and Hopf algebras
- modular (automorphic) forms
- elliptic curves
- the Weyl algebra and its applications
- knots and their invariants

Here is a list of possible topics from a few years ago. There is quite a bit of overlap with my list.

- The fundamental theorem of algebra.
- The Banach-Tarski paradox.
- Brouwer's fixed-point theorem.
- Knot invariants.
- Gödel's incompleteness theorem.
- Ruler and compass constructions.
- Transcendental numbers.
- The prime number theorem.
- Wavelets.
- Fermat's last theorem.
- The four-color theorem.
- The Riemann hypothesis.
- The Poincaré conjecture.

- Hyperbolic groups.
- The classification of finite groups.
- Primality testing.
- Cryptology.
- Random walks and electrical networks.
- Prime-producing polynomials.
- The unsolvability of the word problem for groups.
- Burnside groups.
- The “P=NP” problem.
- Nonstandard analysis.
- The unsolvability of the quintic equation.

In spite of its length, this list is not intended to be exhaustive. I encourage you to look around and come up with your own ideas. One way to do this would be to browse through general books about mathematics, such as Courant and Robbins, *What is mathematics?* (I will put it and other such text on reserve in the math library). You might also get ideas by looking at journals that have survey articles and/or book reviews, such as *The American Mathematical Monthly*, the *Bulletin of the American Mathematical Society*, the *Notices of the American Mathematical Society*, or the *Mathematical Intelligencer*.

Quite a few of the above sources are on-line. As a starting point, go to <http://www.math.cornell.edu/~sjamaar/classes/401/index.html>

**Other resources.** One of the things you’ll learn this semester is how to get information about a mathematical topic. There are obvious sources, such as the library and the internet, but you should also learn to use mathematicians. Don’t be afraid to approach anyone in the department to ask questions or to ask for references. (I’ll provide an introduction if you want.) You’re not imposing when you do this; most mathematicians love to talk about mathematics. You can also contact people not at Cornell (email usually works best).

**Web site.** <http://www.math.cornell.edu/~barbasch/courses/401-06/index.html>

**Feedback.** Please feel free to make suggestions at any time. I am quite willing to experiment.