$W_\delta = W(C_k) \times W(C_{n-k})$ $\qquad W^0_\delta = W(D_k) \times W(C_{n-k})$.

$\underbrace{\nu_1 \geq \nu_2 \geq \cdots \geq \nu_k \geq 0}$ $\qquad \underbrace{\nu_{k+1} \geq \cdots \geq \nu_n \geq 0}$

$$\boxed{\begin{matrix} \delta = (1\ \underbrace{0 \cdots\cdots 0}). & & \delta = (1\ 0 \cdots 0) \\ & \longrightarrow & \\ \nu = (a\ 0 \cdots\cdots 0) & & \nu' = (-a\ 0 \cdots 0). \end{matrix}}$$

$(\nu_1 \cdots \nu_k; \nu_{k+1} \cdots \nu_n) \longrightarrow (\nu_1 \cdots \nu_{k-1}\, -\nu_k; \nu_{k+1} \cdots \overset{\overset{0}{\shortparallel}}{\nu_n})$

Need to choose K-types.

$(2 \cdots 2$

$\nu = (\ a_1 \geq a_2 \geq a_3 \cdots\cdots a_t \,|\, 0 \cdots 0 \,|\, 0 \cdots 0\ )$

$\delta = (\ \varepsilon_1\ \ \varepsilon_2 \cdots\cdots 1 \cdots 1\ 0 \,|\, 0 \cdots 0 \,|\, 1 \cdots 1\ )$

In $Sp(2)$ $\qquad (\nu\ 0) \qquad \delta = (1\ 1)$

LKT's $(1\,1)$ & $(-1\,-1)$ are apart.

Two cases: (1) GL_2 intertwining operators

(2) $\delta = (\varepsilon, 1 \cdots 1)$ $\qquad \delta = (\varepsilon, 1 \cdots 1)$

$\nu = (a, 0 \cdots 0) \longrightarrow \nu = (-a, 0 \cdots 0)$

What K-types?

We need to rewrite the calculation to

bring in w^0_σ. =

$\sigma = (1\ 0)$ $\nu = (\nu_1, \nu_2)$

$\downarrow$

$(0\ 1)$ $\nu = (\nu_2\ \nu_1)$

want these to be scalars independent of ν.

$W_\sigma = W(C_k) \times W(C_{n-k})$

$(\nu\ 0)$. This matches an $A1(SL_2)$

$0 \leq \nu < 1$ unitary.

$$\delta = (\underbrace{1 \cdots 1}_{k} \ \underbrace{0 \cdots 0}_{\ell})$$

$$W_\delta = Sp(k) \times Sp(\ell)$$

Relevant spherical K-types.

$$(\underbrace{1 \cdots 1}_{a} \ 0 \cdots 0 \ \underbrace{-1 \cdots -1}_{a})$$

$$\pm(\underbrace{2 \cdots 2}_{b} \ 0 \cdots 0)$$

Conjectured Relevant K-types for δ:

$$(\underbrace{1 \cdots 1}_{k+a} \ 0 \cdots 0 \ \underbrace{-1 \cdots -1}_{a}) \longleftrightarrow (0 \cdots 0 ; \underbrace{1}_{a} \ 0 \ \underbrace{-1}_{a})$$

$$(\underbrace{2 \cdots 2}_{b} \ \underbrace{1 \cdots 1}_{k} \ 0 \cdots 0) \longleftrightarrow (0 \cdots 0 ; \underbrace{2 \cdots 2}_{b} \ 0 \cdots 0)$$

$$(\underbrace{2 \cdots 2}_{a} \ \underbrace{1 \cdots 1}_{a} \ 0 \cdots 0) \longleftrightarrow (\underbrace{1}_{a} \ 0 \cdots \underbrace{-1}_{a} ; \ 0)$$

$$(\underbrace{1 \cdots 1}_{k-b} \ 0 \cdots 0 \ \underbrace{-1 \cdots -1}_{b}) \longleftrightarrow (0 \cdot \underbrace{-2}_{b} ; \ 0)$$