Euler Method on an Example With Complex Eigenvalues

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Consider the system

\[
\begin{align*}
\dot{x} &= \alpha x + \beta y \\
\dot{y} &= -\beta x + \alpha y \\
\dot{z} &= -\gamma z + Q(x, y)
\end{align*}
\]

where $Q(x,y)$ is the quadratic function $c_{00}x^2 + 2c_{01}xy + c_{11}y^2$

When $c_{01} = 0$ one can show that the unstable manifold through the origin is given exactly by the graph of

\[z = a_{00}x^2 + 2a_{01}xy + a_{11}y^2\]

where

\[
\begin{align*}
a_{01} &= \frac{\beta(c_{11} - c_{00})}{4\beta^2 + \delta^2} \\
a_{00} &= \frac{c_{00} + \beta a_{01}}{\delta} \\
a_{11} &= \frac{c_{00} - \beta a_{01}}{\delta}
\end{align*}
\]

and $\delta = 2\alpha + \gamma$. A closed form formula also exists when $c_{01}$ is nonzero.