

Euler Method on an Example With Complex Eigenvalues

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Consider the system

$$\begin{aligned}\dot{x} &= \alpha x + \beta y \\ \dot{y} &= -\beta x + \alpha y \\ \dot{z} &= -\gamma z + Q(x, y)\end{aligned}$$

where $Q(x,y)$ is the quadratic function $c_{00}x^2 + 2c_{01}xy + c_{11}y^2$

When $c_{01} = 0$ one can show that the unstable manifold through the origin is given exactly by the graph of

$$z = a_{00}x^2 + 2a_{01}xy + a_{11}y^2$$

where

$$\begin{aligned}a_{01} &= \frac{\beta(c_{11}-c_{00})}{4\beta^2+\delta^2} \\ a_{00} &= \frac{c_{00}+\beta a_{01}}{\delta} \\ a_{11} &= \frac{c_{00}-\beta a_{01}}{\delta}\end{aligned}$$

and $\delta = 2\alpha + \gamma$. A closed form formula also exists when c_{01} is nonzero.