TYPES OF INTEGRALS

Each of these gives a number. The function \( f \) is typically some kind of density.

**Double Integral:** \( \int \int_{D} f(x, y) \, dx \, dy \).

*Associated Riemann Sum:* \( \Sigma f(x, y) \Delta x \Delta y \).

*Calculation Method:* Fubini’s Thm. e.g. \( \int_{x=0}^{1} \int_{y=x^3}^{x} xy \, dy \, dx \).

*Changing to polar coordinates often helpful.*

**Triple Integral:** \( \int \int \int_{\Omega} f(x, y, z) \, dx \, dy \, dz \).

*Associated Riemann Sum:* \( \Sigma f(x, y, z) \Delta x \Delta y \Delta z \).

*Calculation Method:* Fubini’s Thm. e.g. \( \int_{x=-2}^{2} \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{z=x^2+y^2}^{8-x^2-y^2} 1 \, dz \, dy \, dx \).

*Changing to spherical/cylindrical coordinates often helpful.*

**Surface Integral of a Function:** \( \int \int_{S} f(x, y, z) \, d\sigma \).

*Associated Riemann Sum:* \( \Sigma f(x, y, z) \Delta \sigma \).

*Calculation Method:*

1. \( \vec{r}(u, v) = < x(u, v), y(u, v), z(u, v) > \)
2. \( \vec{r}_u = < x_u, y_u, z_u >, \vec{r}_v = \ldots \)
3. \( d\sigma = \| \vec{r}_u \times \vec{r}_v \| \, du \, dv \)
4. *Reduce to a double integral.*

**Surface Integral of a Vector Field:** \( \int \int_{S} \vec{E}(x, y, z) \cdot \hat{n} \, d\sigma \).

Exactly the same as \( \int \int_{S} \vec{E}(x, y, z) \cdot \hat{n} \, dS \).

*Associated Riemann Sum:* \( \Sigma (\vec{E}(x, y, z) \cdot \hat{n}) \Delta \sigma \).

*Calculation Method:*

1. \( \vec{r}(u, v) = < x(u, v), y(u, v), z(u, v) > \)
2. \( \vec{r}_u = < x_u, y_u, z_u >, \vec{r}_v = \ldots \)
3. \( \hat{n} = \pm \frac{\vec{r}_u \times \vec{r}_v}{\| \vec{r}_u \times \vec{r}_v \|} \)
4. \( d\sigma = \| \vec{r}_u \times \vec{r}_v \| \, du \, dv \)
5. *Reduce to a double integral.*

In physics, one often has occasion to extend the first three integrals above to vector-valued integrals; e.g. \( \int \int_{D} \vec{F}(x, y) \, dx \, dy \), etc. For example, \( \vec{F} \) might be a vector field describing the force density and the integral would be the total force. Mathematically, this kind of integral is just several ordinary double integrals - one for each component.