CAO CONVENTIONS

Working in local coordinates $\{x_i\}$.

(1) $\partial_i = \dfrac{\partial}{\partial x_i}$, a vector field.
(2) $\nabla_i =$ covariant derivative in the direction ∂_i.
(3) $Rm(X,Y)Z = \nabla_X(\nabla_Y Z) - \nabla_Y(\nabla_X Z) - \nabla_{[X,Y]}Z$.
(4) The second covariant derivative

$$\nabla^2_{X,Y}\alpha = \nabla_X(\nabla_Y\alpha) - \nabla_{\nabla_X Y}\alpha$$

for a 1-form α (or any tensor.)
This $\nabla^2_{X,Y}\alpha$ is also denoted $\nabla_X\nabla_Y\alpha$. At variance with the usual Koszul style way of defining $Rm(X,Y)Z$ without any parentheses!
(5) *Cao:* $R_{ijkl} = g_{km}R^m_{ijl}$. *(Might write as* $R_{ij}{}^m{}_l$ *)*
CLN: $R_{ijkl} = g_{lm}R^m_{ijk}$. *(Might write as* $R_{ijk}{}^m$*)*
(6) $R_{ik} = g^{jl}R_{ijkl}$. *or* $R_{ik} = g^{jl}R^p_{pik}$.*)*
(7) $\nabla_{\partial_i}\partial_j = \Gamma^k_{ij}\partial_k$.
(8) $R^l_{ijk} = \partial_i\Gamma^l_{jk} - \partial_j\Gamma^l_{ik} + \Gamma^l_{ip}\Gamma^p_{jk} - \Gamma^l_{jp}\Gamma^p_{ik}$.