

# Math 105 Prelim #3 – November 29, 2005

This exam has 4 problems and 4 numbered pages. Also, a normal distribution table will be provided.

*You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order in the spaces provided, indicating final answers clearly. (You may use the backs of the pages for scratch paper only) Answers must be justified whenever possible in order to earn full credit. **Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.** Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

*Calculators are permitted, but no other aids are allowed.*

*You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.*

Name (please print): \_\_\_\_\_

Lecture (circle one): Lec 1 (10:10); Lec 2 (11:15); Lec 3 (12:20); Lec 4 (1:25).

Instructor: \_\_\_\_\_

***Please sign the following standard integrity pledge:***

“Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination.”

\_\_\_\_\_

Problem	Score
1	
2	
3	
4	

TOTAL: \_\_\_\_\_

**1.** (25 points) For each of the following matrices, indicate whether or not it can be the transition matrix of a Markov chain and, if so, whether the corresponding Markov chain is regular. Explain your answer.

$$A = \begin{pmatrix} 0 & 1 \\ 1/3 & 2/3 \end{pmatrix}; \quad B = \begin{pmatrix} 1/3 & 2/3 \\ 0 & 1 \end{pmatrix}; \quad C = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 1/4 & 2/3 & 1/12 \\ 1/5 & 3/4 & 1/10 \end{pmatrix}.$$

**2.** (*25 points*) At the Johnson family Thanksgiving, there are 15 place settings. Each setting has a  $\frac{3}{5}$  chance of having an orange napkin and a  $\frac{2}{5}$  chance of having a tan napkin.

**a)** What is the probability that there are exactly 10 orange napkins? (You do not need to simplify your expression.)

**b)** Use the normal distribution to approximate the probability of having at least 7 orange napkins.

**3.** (25 points)

a) Suppose the weights of *ExtraordiNuts<sup>TM</sup>* brand peanuts follow a normal distribution with mean  $\mu = 2.5$  grams and standard deviation  $\sigma = .25$  grams. While splitting a bag of *ExtraordiNuts<sup>TM</sup>*, Bob proposes that he and Alice play the following game: They select a peanut at random from the bag and weigh it. If it weighs more than 3 grams, Bob gives Alice 2 dollars. Otherwise, Alice gives Bob 1 dollar. What are Alice's expected winnings from the game?

b) Suppose Alice, having tired of peanuts, picks ten cards randomly with replacement from a standard 52-card deck. What is the probability that at most 3 of the chosen cards are hearts? (You do not have to simplify your expression.)

**4.** (*25 points*) Each year Aunt Mildred brings either pumpkin cookies or cranberry bread to the Johnson family Thanksgiving. She can never remember what she brought more than one year ago. If she brings pumpkin cookies one year, she has a  $\frac{4}{5}$  probability of bringing cranberry bread the next year. If she brings cranberry bread one year, she has  $\frac{2}{3}$  probability of bringing pumpkin cookies the next year. This determines a regular Markov chain with two states: namely, the first state occurs when Aunt Mildred brings pumpkin cookies; the second when she brings cranberry bread.

**a)** Draw a transition diagram and give the transition matrix for this Markov chain.

**b)** Suppose that Aunt Mildred brings pumpkin cookies this year. What is the probability that she will bring pumpkin cookies two years from now?

**c)** Find the equilibrium vector for this Markov chain without using your calculator.