Math 105 Prelim #3 – November 18, 2004

This exam has 5 problems and 5 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ___________________________________

Instructor: ___________________________________

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TOTAL: ____________

Binomial distribution
If p is the probability of success and n the number of trials, then the probability of exactly x successes is

$$\binom{n}{x} p^x (1-p)^{n-x}.$$

Standard Deviation
The standard deviation of the n numbers $x_1, x_2, x_3, \ldots, x_n$ with mean $\overline{x}$ is

$$s = \sqrt{\frac{\sum x_i^2 - n(\overline{x})^2}{n-1}}.$$

Mean and Standard Deviation for Binomial Distribution
For the binomial distribution, the mean and standard deviation are given by

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}.$$

Expected Value
Suppose the random variable x can take on the n values $x_1, x_2, \ldots, x_n$. Also, suppose the probabilities that these values occur are, respectively, $p_1, p_2, \ldots, p_n$. Then the *expected value* of the random variable is

$$E(x) = x_1p_1 + x_2p_2 + \cdots + x_np_n.$$

The final examination will be held on **Thursday, December 9, 2004** at **9:00 a.m.** in **Goldwin Smith Hall (GS) HEC**.

Please sign below acknowledging that you have read all of the prelim instructions and the final examination information.

(signed) ________________________________

1. (*16 points*)

In a certain Kindergarten class, there is a cage containing 14 gerbils. For a math project, the class weighed each of the gerbils, and found the following weights (in ounces):

$$6, 5, 5, 8, 2, 3, 9, 5, 6, 4, 6, 6, 3, 9.$$

(a) Find the mean and median for this list of weights.

(b) Find the standard deviation for this list of weights.

(c) How many gerbils fall within 1 standard deviation of the mean?

(d) Suppose that the weights in the national gerbil population are distributed **normally**. If our sample of 14 gerbils were typical, how many would we expect to fall within 1 standard deviation of the mean?

2. (*26 points*) Two people, A and B, play the following game. They draw two balls from a jar that has five balls numbered 1, 2, 3, 4 and 5. Let n be the sum of the numbers on each of the two balls drawn. If n is even, A pays n dollars to B. If n is odd, B pays n dollars to A.

(a) Let x be the random variable representing the net profit of player A. (Note that the profit could be negative.) Draw a table showing the probability distribution of x.

(b) What is the expected value of x? If you could choose whether to play as A or as B, which would you choose?

3. (*16 points*) A company that makes wine glasses has determined that the probability that a glass is defective is 5%.

(a) Estimate, to 4 decimal places, the probability that in a randomly selected sample of 1000 glasses at least 55 are defective.

(b) Estimate, to 4 decimal places, the probability that in a randomly selected sample of 1000 glasses between 50 and 55 (inclusive) are defective.

4. (*18 points*) The average wingspan of a certain type of bird is 12.1 in., with a standard deviation of 3 in. Assume that the length of the wingspan is normally distributed.

(a) Find the probability that a randomly chosen bird has a wingspan greater than 14 in.

(b) Find the probability that a randomly chosen bird has a wingspan between 11 in. and 13 in.

(c) Find the length so that 30% of birds have a wingspan greater than that length.

5. (*24 points*) Do not use the normal distribution to answer this question.

On any given morning, the probability that Henri wakes up with a hangover is 3/4.

(a) What is the probability that Henri wakes up with a hangover exactly three mornings in a given week?

(b) What is p, the probability that Henri wakes up with a hangover at most four mornings in a given week?

(c) Using the answer to part (b), find the probability that three weeks in a row, Henri wakes up with a hangover at most four mornings in each week?

(d) Recall that there are 52 weeks in a year. What is the probability that in exactly nine weeks of the year, Henri wakes up with a hangover at most four times that week?