Math 105 Prelim #2 – October 27, 2005

This exam has 5 problems on 5 pages, preceded by this cover page and a page with two formulas.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order in the spaces provided, indicating final answers clearly. (You may use the backs of the pages for scratch paper only) Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name (please print): ______________________________

Instructor: ______________________________

Please sign the following standard integrity pledge:

"Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination."

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Problem	Score
1	
2	
3	
4	
5	

TOTAL: __________

Two formulas that may be useful

Bayes' Theorem:

$$P(F_i|E) = \frac{P(F_i) \cdot P(E|F_i)}{P(F_1) \cdot P(E|F_1) + P(F_2) \cdot P(E|F_2) + \ldots + P(F_n) \cdot P(E|F_n)}.$$

Bayes' Theorem (Special Case):

$$P(F|E) = \frac{P(F) \cdot P(E|F)}{P(F) \cdot P(E|F) + P(F') \cdot P(E|F')}.$$

1. (*15 points*) Suppose the organizer of a film festival wishes to show three romantic comedies and three westerns.

a) How many orders of the movies are possible?

b) How many orders are possible if the organizer wishes to alternate between genres (i.e., if a western is shown, the next movie shown is a romantic comedy, and so on)?

c) How many orders are possible if the only restriction on the order is that the last movie shown has to be a western?

2. (*20 points*) Bengt and his friends go hiking every weekend. The probability that they remember to take a map along is 2/3. The probability that they *both* take a map along *and* get lost is 1/6.

a) Given that they remembered the map, what is the conditional probability that they got lost?

b) If they forget the map, they have a 3/4 chance of not getting lost. Given that Bengt and his friends did not get lost last weekend, what is the probability that they took a map along?

c) Are the event that they remember the map and the event that they get lost independent events? Explain why or why not.

d) What is the probability that Bengt and his friends got lost? What is the probability that they either got lost or forgot the map?

3. (*15 points*) Five of your friends were born in the month of October (which has 31 days.)

a) What is the probability that no one shares the same birthday?

b) What is the probability that at least two friends share the same birthday?

c) What is the probability that at most 4 have the same birthday?

4. (*20 points*) Ping pong balls numbered 1 through 20 are placed in a tumbler, and two are drawn out randomly in succession without replacement. Let the number on the first ball be denoted by m and the number on the second ball denoted by n. Suppose that E stands for the event $5 \le m \le 12$ and F for the event $5 \le n \le 12$.

a) Draw a tree diagram illustrating this experiment. Make sure to label the nodes of the tree carefully, and fill-in the probabilities associated with the edges of the tree.

b) Calculate $P(F)$.

c) Calculate $P(E'|F)$.

d) Determine whether or not E and F are independent, using the definition of independence of events.

5. (*30 points*) **a)** Five friends, Alice, Bob, Cynthia, David, and Ed get together to play a card game called Shark's Quarry. The game is played with the usual 52 card deck. It starts with each player being dealt five cards. A player's hand is called a "Deep Six" if he or she holds only one six, and that six is the highest card in the hand. What is the probability that Alice is dealt a Deep Six? (The only denominations lower than a six are two, three, four, and five.)

b) When the game is ends, the friends decide to watch TV, choosing seats randomly in a row on a long sofa. What is the probability that Alice and Bob end up sitting together?

c) With the same five friends sitting randomly on the sofa as in problem 5 b), what is the probability that Cynthia is sitting between Alice and Bob? (Note: We don't care if Cynthia is adjacent to Alice or to Bob, just that she is sitting between them. Part c) may be a bit more challenging than the other two parts, so don't spend an excessive amount of time on it.)