Math 105 Prelim #1 – September 27, 2005

This exam has 6 problems and 7 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order in the spaces provided, indicating final answers clearly. (You may use the backs of the pages for scratch paper only) Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name (please print): _____

Instructor: _____

Please sign the following standard integrity pledge:

"Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination."

_____

Problem	Score
1	
2	
3	
4	
5	
6	

TOTAL: _____

1. (*16 points*) In a Cornell calculus class of 120 students

43 come from the mid-west;
42 live off-campus;
5 come from the mid-west, live off-campus, and are pre-med;
13 come from the mid-west and are pre-med;
15 come from the mid-west and live off-campus;
12 live off-campus and are pre-med;
20 do not come from the mid-west, do not live-off campus, and are not pre-med.

(a) Draw a Venn diagram that illustrates the above sets of students, carefully labeling each part of the diagram. (You will use this diagram to answer other parts of the problem.)

(b) How many students come from the mid-west and are pre-med, but do not live off-campus?

(c) How many students come from the mid-west but are not pre-med and do not live off campus?

(d) How many students are pre-med?

2. (*16 points*)

(a) Rachel is raising money for the Red Cross. She has committed to walk 13 miles if she receives $800 in donations. For each additional $10 donated, she will add 0.2 miles to her walk. How much must she get to walk a total of 15 miles?

(b) Letting d stand for the number of dollars raised by Rachel, and w for the number of miles Rachel walks, give a linear equation expressing w in terms of d.

(c) How much money needs to be raised for Rachel to walk a marathon (26 miles)?

3. (*16 points*)

(a) Let ℓ denote the line in the xy-plane given by the equation $5y - x = 4$. Give the equation of the line in the xy-plane perpendicular to ℓ and passing through the point $(-1, 1)$. Sketch ℓ and the line perpendicular to it in the xy-plane.

(b) Your friend wants to analyze two sets of data he has collected. He has computed that the coefficient of correlation for the first set is 0.2 and the coefficient of correlation for the second set is -0.9. For which set would you recommend that he use a least squares line to model his data (which one, neither, or both)? Explain.

4. (*16 points*) Suppose that A is a 4×2 matrix and $B = \begin{bmatrix} 1 & 6 & 7 \\ 3 & -2 & 5 \end{bmatrix}$.

(a) Can one add matrix A and matrix B? Explain.

(b) Is AB defined? Explain.

(c) Let $B = \begin{bmatrix} 1 & 6 & 7 \\ 3 & -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 8 \\ -1 & 5 \\ 4 & 2 \end{bmatrix}$, and $D = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$.

Compute the matrix $BC - D$.

5. (*20 points*)

(a) Compute the inverse of the matrix

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 3 \\ 0 & 2 & 3 \end{bmatrix}.$$

(b) Suppose that you have calculated that the inverse of the matrix $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$ is given by $B^{-1} = \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$. Use this to solve each of the following two equation systems:

$$\begin{aligned} 2x + 5y &= 1 \\ 3x + 7y &= 5 \end{aligned} \qquad\qquad \begin{aligned} 2x + 5y &= 100 \\ 3x + 7y &= 100. \end{aligned}$$

6. (*16 points*) Erin's bookcase contains 120 books, of which 80 are textbooks and 80 are hardcover. (Some books may be neither.) If we look at just her textbooks, we see that 3/4 of these are hardcover. Erin randomly selects one of the books on her shelf (i.e., each book has an equal likelihood of being selected).

(a) Regard Erin's selection procedure as an "experiment", in the sense discussed in class and in the textbook. Let S be the sample space for this experiment, let E_1 denote the event described by "the selected book is a textbook," and let E_2 denote the event described by " the selected book is hardcover." Describe the sets S, E_1, E_2, and draw a Venn diagram for these. (You may use this diagram to help you with the other parts of the problem.)

(b) Calculate the probability that Erin does *not* select a textbook.

(c) Calculate the probability that she picks a hardcover textbook.

(d) Calculate the probability that the book she picks is either hardcover or a textbook.