Problem 1 (16pts)

(a) Let $C_A(x) = mx + b$ denote the cost function of publishing house A. Then $C_A(10) = 30$ and $C_A(40) = 90$. Hence

$$\begin{cases} 10m + b = 30 \\ 40m + b = 90. \end{cases}$$

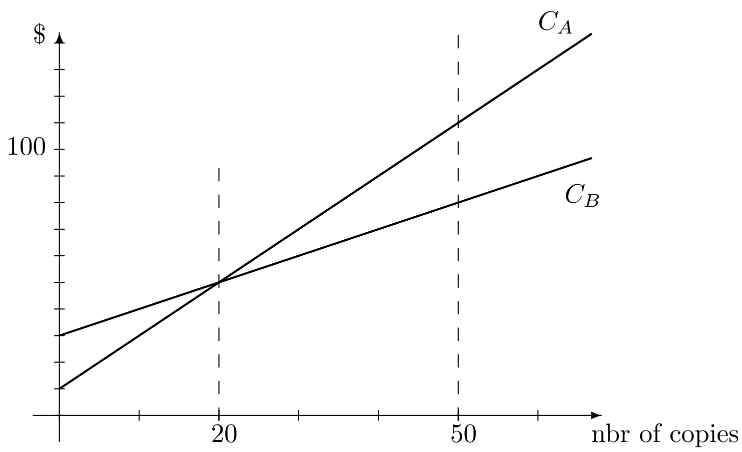
Multiplying the first equation by 4 and substracting the second second equation gives $3b = 30$, that is, $b = 10$. Reporting this value in the first equation gives $10m = 20$, i.e., $m = 2$. Thus the cost function of publishing house A is

$$C_A(x) = 2x + 10$$

(b) Let $C_B(x) = mx + b$ denote the cost function of publishing house B. Then the slope m is the marginal cost so that $m = 1$. Moreover, $50m + b = 80$. As $m = 1$, this gives $b = 30$. Thus the cost function of publishing house B is

$$C_B(x) = x + 30.$$

(c)

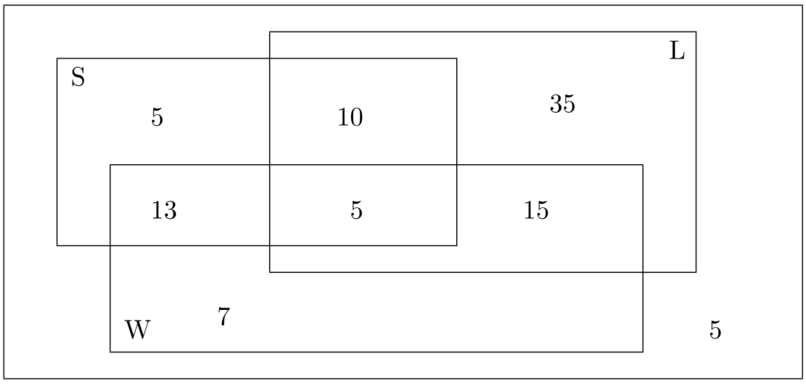


To print 50 copies, publishing house B is the cheaper choice with a cost of \$ 80.

(d) The cost are the same if $C_A(x) = C_B(x)$, that is $2x + 10 = x + 30$. This gives $x = 20$. Printing 20 copies costs the same (\$ 50) at publishing houses A and B.

Problem 2 (15pts)

(a)



(b) The number of students having a scholarship, working on campus but having no student loan is $18 - 5 = 13$.

(c) The number of students having a student loan but no scholarship and who do not work on campus is 35.

(d) $33 = 5 + 5 + 10 + 13$ Students have a scholarship.

Problem 3 (15pts)

We write the system of equation in standard form and use the echelon method.

$$\left\{\begin{array}{rcrcrcr} x & + & 3y & - & 6z & = & 7 \\ 2x & - & y & + & 2z & = & 0 \\ x & + & y & + & 2z & = & -1 \end{array}\right.$$

We replace (R_2) by $(R_2) - 2(R_1)$ and (R_3) by $(R_3) - (R_1)$.

$$\left\{\begin{array}{rcrcrcr} x & + & 3y & - & 6z & = & 7 \\ & - & 7y & + & 14z & = & -14 \\ & - & 2y & + & 8z & = & -8 \end{array}\right.$$

We replace (R_2) by $-\frac{1}{2}(R_2)$ and then (R_3) by $(R_3) + 2(R_2)$.

$$\left\{\begin{array}{rcrcrcr} x & + & 3y & - & 6z & = & 7 \\ & & y & - & 7z & = & 7 \\ & & & & 4z & = & -4 \end{array}\right.$$

Thus $z = -1$, $y + 7 = 7$, that is, $y = 0$, and $x + 6 = 7$, that is, $x = 1$. The solution of the system is $x = 1$, $y = 0$, $z = -1$.

Problem 4. (14pts)

(a)

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \overset{(R_2)-2(R_1)}{\longrightarrow} \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right) \overset{(R_1)+3(R_2)}{\longrightarrow} \left(\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & -1 & -2 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & -1 & -2 & 1 \end{array}\right) \overset{-(R_2)}{\longrightarrow} \left(\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right).$$

The inverse of A is

$$A^{-1} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}.$$

(You should verify that $AA^{-1} = I$)

(b) Let $X = \binom{x}{y}$, $B = \binom{-2}{6}$ To solve $AX = B$, we multiply by A^{-1} to get $X = A^{-1}B$. We have $A^{-1}B = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}\binom{-2}{6} = \binom{28}{-10}$. Thus $x = 28$, $y = -10$ is the solution.

Problem 5 (12pts)

(a) It is not possible to form $A + B$ because A and B do not have the same shape.

(b) The matices B and C have the same shape and $B + C$ exists.

(c) $B + C = \begin{pmatrix} 2 & 1 \\ 0 & 1 \\ 5 & 2 \end{pmatrix}$.

(d) For the product AB to be defined, the number of columns of A must equal the number or rows of B. This is not the case so AB does not exists.

(e) The number or columns of B equal the number of rows of A. We can compute BA.

(f) $BA = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 6 \end{pmatrix}$.

Problem 6 (12pts)

(a)

$$S = \{(B,B,B),(B,B,G),(B,G,B),(G,B,B),\\ (B,G,G),(G,B,G),(G,G,B),(G,G,G)\}.$$

(b)

$$E = \{(B,G,G),(G,B,G),(G,G,B),(G,G,G)\}.$$

(c) As each composition is equally likely and there are 8 distinct family compositions, any one has probability $1/8$. It follows that $P(E) = 4/8 = 1/2$.

Problem 7 (16pts)

(a) Reading the text gives

$$P(F) = 1/2, \quad P(H) = 2/3, \quad P(F \cap H') = 1/5.$$

It follows that $P(F') = 1 - P(F) = 1/2$ and $P(H') = 1 - P(H) = 1/3$.

(b) The event "the wuzzle has fur and horns" is $F \cap H$. We have $P(F) = P(F \cap H) + P(F \cap H')$ because $F \cap H$ and $F \cap H'$ are disjoint events. Thus

$$P(F \cap H) = P(F) - P(F \cap H') = (1/2) - (1/5) = 3/10.$$

(c) The event "the wuzzle has either fur or horns" is $F \cup H$ and $P(F \cup H) = P(F) + P(H) - P(F \cap H)$. Thus

$$P(F \cup H) = (1/2) + (2/3) - (3/10) = 13/15.$$

(d) To answer this question, let us compute the probability that a wuzzle has no fur and no horns, that is $P(F' \cap H')$. We have $P(H') = P(F' \cap H') + P(F \cap H')$. Hence

$$P(F' \cap H') = P(H') - P(F \cap H') = (1/3) - (1/5) = 2/15.$$

As $P(F' \cap H') > 0$, we deduce that (under the hypotheses of this exercise) wuzzles with no fur and no horns exist.