Math 105 Prelim #1 – September 28, 2004

This exam has 7 problems and 7 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: _____

Instructor: _____

Problem	Score
1	
2	
3	
4	
5	
6	
7	

TOTAL: _____

1. (*16 points*)

(a) The manager of publishing house A has determined that the cost of producing 10 copies of a certain book is \$30, and that the cost of producing 40 copies of the same book is \$90. Assuming that the cost function is linear, find the cost function for publishing house A.

(b) The manager of publishing house B has determined that its marginal cost is \$1 per book and that the cost of producing 50 copies of the same book is \$80. Assuming that the cost function is linear, find the cost function for publishing house B.

(c) Graph both cost functions on the same axes. Which publishing house would you choose to print 50 copies?

(d) Find the number of copies for which both publishing houses have the same publishing cost.

2. (*15 points*) In a Calculus class of 95 students:

65 have a student loan
40 work on campus
5 have a scholarship, a student loan and work on campus
20 have a student loan and work on campus
15 have a scholarship and a student loan
18 have a scholarship and work on campus
5 do not work on campus, have no student loan and no sholarship.

(a) Draw a Venn diagram and use it to answer the following questions.

(b) How many students have a scholarship, work on campus and have no student loan?

(c) How many students have a student loan but neither work on campus nor have a scholarship?

(d) How many students have a scholarship?

3. (*15 points*) Solve the system of equations

$$\begin{aligned} 3y - 6z &= 7 - x \\ 2z - y + 2x &= 0 \\ x + y + 2z &= -1. \end{aligned}$$

4. (*14 points*) Let A be the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.

(a) Compute A^{-1}.

(b) Find x and y such that $A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$.

5. (*12 points*) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 4 & 1 \end{pmatrix}$.

(a) Is the sum $A + B$ defined?

(b) What about $B + C$?

(c) Compute whichever of (a) or (b) is possible.

(d) Is the product AB defined?

(e) What about BA?

(f) Compute whichever of (d) or (e) is possible.

6. (*12 points*) In this exercise we consider the different possible compositions of three-child families. For instance, (B,G,B) describes a family whose older and younger children are boys and whose middle child is a girl.

(a) Write down the sample space S describing all possible three-child family compositions.

(b) Write down the event $E =$ "the family has at least two girls".

(c) Assuming that all possible family compositions are equally likely, calculate the probability of E.

7. (*16 points*) A wuzzle is a kind of creature. 1/2 of all wuzzles have fur. 2/3 of all wuzzles have horns. 1/5 of all wuzzles have fur but no horns. Now, we perform an experiment by selecting a wuzzle at random and recording its characteristics. Let F be the event "the wuzzle has fur", and let H be the event "the wuzzle has horns".

(a) Find $P(F)$, $P(F')$, $P(H)$, $P(H')$, and $P(F \cap H')$.

(b) What is the probability of the event "the wuzzle has fur and horns"?

(c) What is the probability of the event "the wuzzle has either fur or horns"?

(d) Do wuzzles with no fur and no horns exist?