Quiz on Derivatives

Friday 14th, October

Name: __________________________
Net ID: ________________________
Section: ________________________

1. State the definition of the derivative of a function \( f(x) \). (2pts)

The derivative is a function

\[ f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \]

It's the slope of point \( x \) at the intersection with the graph of \( f(x) \).

2. Compute the following derivatives using any tricks you know. (2pts each)

(a) \( f(x) = 3x^2 + 5x - 12 \)

\( f'(x) = 6x + 5 \)

Using the power rule \((x^n)' = nx^{n-1}\), we get \((3x^2 + 5x - 12)' = 3 \cdot 2x + 5 \cdot 1 = 6x + 5\)

(b) \( f(x) = 2e^x \)

\( f'(x) = 2e^x \)

Using the power rule, \((2e^x)' = 2e^x\) Answer: 0.

(c) \( f(x) = \sin(x) \cdot \ln(x) \)

Product rule: \((fg)' = f'g + fg'\)

\[-\cos(x) \cdot \ln x + \sin(x) \cdot \frac{1}{x} = \]

\(= (\sin x - \cos x) \cdot \ln x \)

(d) \( f(x) = \ln(\cos(x)) \)

\( f'(x) = \frac{1}{\cos x} \cdot (-\sin x) \)

\( f'(x) = \frac{-\sin x}{\cos x} \) ?
(e) \( f(x) = \frac{e^x}{x} \)

\[
\left( \frac{e^x}{x} \right)' = \frac{x \cdot e^{x-1} - e^x}{x^2} = e^{x-1} \]

Using the quotient rule,

\[
\left( \frac{e^x}{x} \right)' = e^x \cdot \left( \frac{1}{x} \right) + e^x \cdot \left( \frac{1}{x} \right)' \cdot \frac{1}{x} = e^x \left( -\frac{1}{x^2} + \frac{1}{x} \right) = \frac{e^x (x-1)}{x^2}
\]

3. Let \( f(x) \) be a function, \( a \) some real number, and \( h \) a variable. Decide what best represents the following expressions (delete as appropriate), and use a few of your own words to describe the expression given. (2pts each)

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>Function/Number</th>
<th>Function/Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(a) )</td>
<td>Function/Number</td>
<td>Number</td>
</tr>
<tr>
<td>The slope of ( f(x) ) at ( a )</td>
<td>Function/Number</td>
<td>Slope ?</td>
</tr>
<tr>
<td>The tangent line to ( f(x) ) at ( a )</td>
<td>Function/Number</td>
<td>Line at a point ( f(x) )</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>Function/Number</td>
<td>Number</td>
</tr>
<tr>
<td>( f'(a) )</td>
<td>Function/Number</td>
<td>Number</td>
</tr>
<tr>
<td>( \frac{f(x+h)-f(x)}{h} )</td>
<td>Function/Number</td>
<td>Number</td>
</tr>
</tbody>
</table>

4. True or false: \( \frac{d}{dx} [\ln(\pi)] = \frac{1}{\pi} \). (2pts)

Since \( (\ln x)' = \frac{1}{x} \), get

\[ \ln(\pi)' = \frac{1}{\pi} \cdot \pi = 1 \] True

5. Using the chain rule, show that the derivative of \( \ln(x) \) is \( 1/x \). That is, start with the composition

\[ e^{\ln(x)} = x \]

and differentiate it using the chain rule. Do not use implicit differentiation, and do not use the formula for the derivative of the inverse function. (6pts)

\[ e^{\ln(x)} = x \]

Derivative:

\[
\left( e^{\ln(x)} \right)' = x' = 1
\]

\[
\left( e^{\ln(x)} \right)' = \ln(x) \cdot e^{\ln(x)} \cdot (\ln x) = \ln(x) \cdot \frac{e^{\ln(x)}}{\ln x} \cdot (\ln x) = e^{\ln(x)} \cdot (\ln x)' = x \cdot (\ln x)' = 1
\]

So

\[ (\ln x)' = \frac{1}{x} \]