Surfaces. Embedded graphs

Sasha Patotski
Cornell University
ap744@cornell.edu

November 18, 2014
Basic surfaces

**Definition**

A (closed) *surface* $\Sigma$ is a geometric figure, or a subset of $\mathbb{R}^n$, such for any point $x \in \Sigma$ there exists a small neighbourhood $U$ of $x$ in $\Sigma$ which is homeomorphic to an open disc in $\mathbb{R}^2$.

**In other words:** a surface is something that looks locally like a plane.
Definition

A (closed) surface $\Sigma$ is a geometric figure, or a subset of $\mathbb{R}^n$, such for any point $x \in \Sigma$ there exists a small neighbourhood $U$ of $x$ in $\Sigma$ which is homeomorphic to an open disc in $\mathbb{R}^2$.

In other words: a surface is something that looks locally like a plane.

Plane $\mathbb{R}^2$, sphere $S^2$, torus $T^2$, Möbius band $M^2$, Klein bottle $K^2$, projective plane $\mathbb{R}P^2$. 
Basic surfaces

Definition

A (closed) surface $\Sigma$ is a geometric figure, or a subset of $\mathbb{R}^n$, such that for any point $x \in \Sigma$ there exists a small neighborhood $U$ of $x$ in $\Sigma$ which is homeomorphic to an open disc in $\mathbb{R}^2$.

In other words: a surface is something that looks locally like a plane.

Plane $\mathbb{R}^2$, sphere $S^2$, torus $T^2$, Möbius band $M^2$, Klein bottle $K^2$, projective plane $\mathbb{R}P^2$. 
Which surface is that?
What about this one?
The same question here
Try to guess the question
What happens if we glue two Möbius bands together?
Can you cut a Klein bottle into a single Möbius band?
Attaching a handle to a surface
How to get a sphere with $g$ handles out of a polygon?
Non-planar graphs $K_5$ and $K_{3,3}$

**Euler formula:** For any planar graph $G$ one has $V - E + F = 2$. 
Non-planar graphs $K_5$ and $K_{3,3}$

**Euler formula:** For any planar graph $G$ one has $V - E + F = 2$.

Prove that $K_5$ and $K_{3,3}$ are not planar.
Non-planar graphs $K_5$ and $K_{3,3}$

**Euler formula:** For any planar graph $G$ one has $V - E + F = 2$.

Prove that $K_5$ and $K_{3,3}$ are not planar.
Can $K_5$ and $K_{3,3}$ be embedded into a Möbius band?