Surfaces

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Definition

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- $\mathbb{R}^2$ as well as any open subset of it is a surface.
- Sphere: any surface homeomorphic to $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$. 


More surfaces

Torus:
Möbius band and projective plane
Möbius band and projective plane
Projective plane

(a)  

(b)  

(c)  

(d)
Which surface is that?
What about this one?
The same question here
Try to guess the question

\[ \text{Diagram with labeled edges: } a_1, \overline{a}_1, b_1, \overline{b}_1, a_2, \overline{a}_2, b_2, \overline{b}_2 \]
What happens if we glue two Möbius bands together?