

Math 433, Homework 8, Due October 22, 2006 at 2:00pm

From Curtis:

§23 2, 4, 6 (for this problem, assume V is over a field \mathbb{F} in which $1 + 1 \neq 0$); §24 1, 4, 5, 8;
§25 2, 4

Also:

- 1) Find, up to similarity, all 2×2 matrices A over \mathbb{C} such that $A^4 = I$ and $A, A^2, A^3 \neq I$.
- 2) a) Let A and B be 2×2 non-scalar matrices over an algebraically closed field \mathbb{F} . Prove that A and B are similar if and only if they have the same characteristic polynomial.
b) Is the same statement true for 3×3 matrices? Prove it or find a counterexample.
- 3) Let A be $n \times n$ with eigenvalues $\lambda_1, \dots, \lambda_n$. For integers $k > 0$, what are the eigenvalues of A^k ? What is the trace of A^k ?
- 4) Determine necessary and sufficient conditions for a $n \times n$ matrix A over \mathbb{C} to have a “square root”, i.e. conditions so there exists a matrix B over \mathbb{C} such that $B^2 = A$.